S2 Appendix

Optimal density maximizing alternative quality functions.

We considered alternative quality functions combining $E_g$, $E_l$ and $\rho$. For each new $J$, we checked analytically the existence of an optimal density maximizing $J$ in both regular lattices and random networks. We excluded the quality functions for which we obtained indefinite, non-closed or trivial solutions.

- $J = E_g + E_l - \rho \simeq 1/L + C - \rho$
  
  By substituting the expression of $L$ and $C$ for random networks we obtain:
  \[
  J_1 = \frac{\log k}{\log n} + \frac{k}{n} - \frac{k}{n-1}
  \]  
  \hspace{1cm} (S5)

  For the sake of simplicity we solved with respect to $k$, knowing that $\rho = k/(n-1)$. When derivating and equating to zero we had $k = n(n-1)/\log n$, which leads to the impossible condition $\rho = n/\log n > 1$.

- $J = E_g - \rho \simeq 1/L - \rho$
  
  By using the expression of $L$ for lattices we obtain:
  \[
  J_0 = \rho \left( \frac{2(n-1)}{n} - 1 \right)
  \]  
  \hspace{1cm} (S6)

  When derivating and equating to zero we had $n = 2$ and we cannot solve with respect to $\rho$.

- $J = E_l - \rho \simeq C - \rho$
  
  By substituting the expression of $C$ for random networks we obtain:
  \[
  J_1 = \frac{k}{n} - \frac{k}{n-1}
  \]  
  \hspace{1cm} (S7)

  When derivating and equating to zero we had $1/n = 1/(n-1)$ and we could not solve with respect to $\rho$.

- $J = E_g/E_l/\rho \simeq (C/L)/\rho$
  
  By substituting the expression of $L$ and $C$ for random networks we obtain:
  \[
  J_1 = \frac{n-1}{n \log n} \log k
  \]  
  \hspace{1cm} (S8)

  When derivating and equating to zero we had the trivial solution $k = 0$.

Similarly, it is easy to prove that neither $J = E_g/\rho$ nor $J = E_l/\rho$ admitted meaningful solutions in lattices or random networks.