The velocity and spreading rate for the simple model

Recall that, in the simple model, flux is given by \( \phi = pa_n - 1 + q(a_n - 1 - 2a_n) \). Assuming there is no creation or destruction of auxin within cells, this implies

\[
L \frac{da_n}{dt} = p(a_{n-1} - a_n) + q(a_{n-1} + a_{n+1} - 2a_n),
\]

where \( L \) is the length of a cell. If the distribution describes a pulse, then the distance, \( \mu \), that its mean travels in time \( t \) is given by

\[
\mu = \sum_n nL a_n / \sum_n a_n
\]

(S1)

and the velocity, \( v \), is the derivative of the mean:

\[
v = d\mu/dt = \left( \sum_n nL da_n / dt \right) / \sum_n a_n
\]

\[
= \sum_n \left[ p(a_{n-1} - a_n) + q(a_{n-1} + a_{n+1} - 2a_n) \right] / \sum_n a_n,
\]

\[
= p \left[ \sum (n-1)a_{n-1} + \sum a_{n-1} - \sum na_n \right] / \sum a_n
\]

\[
+ q \left[ \sum (n-1)a_{n-1} + \sum a_{n-1} + \sum (n+1)a_{n+1} - \sum a_{n+1} - 2n \sum a_n \right] / \sum a_n,
\]

\[
= p \sum a_n / \sum a_n,
\]

(S2)

which is Eq (2).

To compute the spreading rate, \( \rho \), recall that this is defined to be

\[
\rho = d(Var)/dt,
\]

where the variance is given by:

\[
Var = \frac{\sum_n (nL - \mu)^2 a_n}{\sum_n a_n} = \frac{\sum_n n^2 L^2 a_n}{\sum_n a_n} - \mu^2.
\]

(S3)

We have

\[
d/dt \sum n^2 L^2 a_n = \sum n^2 L^2 da_n / dt
\]

\[
= L \sum n^2 \left[ p(a_{n-1} - a_n) + q(a_{n-1} + a_{n+1} - 2a_n) \right]
\]

\[
= L(p+q) \sum [(n-1)^2 + 2(n-1) + 1]a_{n-1} - L(p+2q) \sum n^2 a_n
\]

\[
+ Lq \sum [(n+1)^2 - 2(n+1) + 1]a_{n+1},
\]

\[
= 2Lp \sum na_n + L(p+2q) \sum a_n,
\]

where the last step follows if we ignore end effects, so

\[
\sum (n-1)a_{n-1} = \sum (n+1)a_{n+1} = \sum na_n
\]

\[
\sum (n-1)^2 a_{n-1} = \sum (n+1)^2 a_{n+1} = \sum n^2 a_n.
\]

Thus

\[
\rho = \frac{d(Var)}{dt} = \frac{\sum n^2 L^2 da_n / dt}{\sum a_n} - 2\mu \frac{d\mu}{dt}
\]

\[
= 2p\mu + L(p+2q) - 2p\mu
\]

\[
= L(p+2q),
\]

(S4)
which is Eq (3).

One can also write down a continuous version of the model:

\[
\frac{\partial a}{\partial t} = -p \frac{\partial a}{\partial x} + q \frac{\partial^2 a}{\partial^2 x},
\]

(S5)

which has a gaussian solution

\[
a = \frac{1}{\sqrt{2\pi qt}} e^{-\frac{(x-\mu)^2}{2qt}}.
\]

(S6)

This explains why a gaussian is such a good approximation in the discrete case.