**Supplemental Text S1**

**Orientational Order Parameter (OOP)**

The OOP characterizes the order of orientation of a single construct. For disordered systems the OOP is zero and for perfectly aligned systems it is one. The OOP is calculated by using a set of vectors, $\vec{p}_i$, and forming a tensor for each of the vectors. The mean tensor is:

$$T = \left\langle \begin{bmatrix} p_{i,x}p_{i,x} & p_{i,x}p_{i,y} \\ p_{i,x}p_{i,y} & p_{i,y}p_{i,y} \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right\rangle = \{\text{Mean tensor}\}. \tag{1}$$

The OOP is defined as the maximum eigenvalue of the mean tensor

$$OOP = \max \left[ \text{eigenvalue}(T) \right] = \{\text{Orientational order parameter}\}$$

$$= \left\langle 2(\vec{p}_i \cdot \hat{n}_p)^2 - 1 \right\rangle = \left\langle \cos(2(\alpha - \alpha_0)) \right\rangle \tag{2}$$

where $\hat{n}_p$ and $\alpha_0$ are the director and mean angle, respectively.

**Symmetry of OOP**

The OOP also has pseudo-vector symmetry and this can be easily shown. To check for symmetry we need to vary the sign of $\vec{p}_i$ and $\hat{n}_p$. If we change the sign of $\vec{p}_i$, $\hat{n}_p$ or both we obtain:

$$\left\langle 2\left\{ -\vec{p}_i \cdot \hat{n}_p \right\}^2 - 1 \right\rangle = \left\langle 2\left\{ \vec{p}_i \cdot (-\hat{n}_p) \right\}^2 - 1 \right\rangle = \left\langle 2\left\{ -\vec{p}_i \cdot (-\hat{n}_p) \right\}^2 - 1 \right\rangle = \left\langle 2\left\{ \vec{p}_i \cdot \hat{n}_p \right\}^2 - 1 \right\rangle. \tag{3}$$

Thus, we will produce the same OOP no matter the sign of $\vec{p}_i$ and $\hat{n}_p$, therefore the OOP is symmetric.

**Second order correlations**

The OOP is not able to characterize second order correlations. To prove this define $P$ as:

$$\vec{p}_i = \left[ \cos\left(\frac{\pi}{2}\right), \sin\left(\frac{\pi}{2}\right) \right] \text{ and } \vec{p}_{i+n} = \left[ \cos(-\frac{\pi}{2}), \sin(-\frac{\pi}{2}) \right] \tag{4}$$
for \( i = 1, ..., n \). Thus, \( \hat{\alpha}_p = 0 \) and \( \alpha_0 = 0 \):

\[
OOP_p = \sum_{i=1}^{2n} \cos(2\alpha) = n \cdot \cos \left( \frac{2\pi}{2} \right) + n \cdot \cos \left( 2 \left( -\frac{\pi}{2} \right) \right) = 0. \quad (5)
\]

Thus, OOP=0 even though there is obvious organization in P.

**Circular Statistics (assume period of \( \pi \))**

It is possible to show that the R of circular statistics [1] is the same as the OOP. If the data is distributed:

\[
\alpha = \frac{2\pi x}{k} \quad (6)
\]

where, \( x \) is the data in the original scale, \( k \) is the total number of steps on the \( x \) scale, and \( \alpha \) is the variable on the new directional scale (i.e. with a standard \( 2\pi \) period). In our case a rod that is \( \beta \) degrees away from the director is physically the same rod as the one \( \beta + \pi \) degrees away. Therefore in our case \( k = \pi \) and:

\[
\alpha = 2\theta \quad (7)
\]

where, \( \theta \) is defined as the angle that we measured from the director. From this it follows:

\[
S = \frac{1}{N} \sum_{i=1}^{N} \sin 2\theta_i \quad (8)
\]
\[
C = \frac{1}{N} \sum_{i=1}^{N} \cos 2\theta_i \quad (9)
\]
\[
R = \sqrt{S^2 + C^2}. \quad (10)
\]

If we assume that the director is orientated such that \( \theta_n = 0 \), then the angles are evenly distributed between positive and negative and therefore \( S = 0 \). We can then write \( R \) as:

\[
R = C = \frac{1}{N} \sum_{i=1}^{N} \cos 2\theta_i = \langle \cos 2\theta_i \rangle. \quad (11)
\]
Therefore, by definition of the director we will have the range $0 < R < 1$ and it is equivalent to the

$$OOP = 2\langle \cos^2 \theta_i \rangle - 1 = \langle \cos 2\theta_i \rangle = R.$$  

**Circular Correlation**

In the special case, with both constructs having a uniform distribution, i.e. both being perfectly isotropic, the correlation coefficient and COOP converge to the same equation. If the angles are uniformly distributed on the circle the correlation coefficient [2] can be written as

$$r = \sqrt{\left(\frac{1}{N} \sum_{i=1}^{N} \cos 2\theta_i\right)^2 + \left(\frac{1}{N} \sum_{i=1}^{N} \sin 2\theta_i\right)^2}$$  \hspace{1cm} (12)

where $\theta$ represents the angle between two biological constructs. If the director is to be assumed $\hat{n} = [1,0]$ then $\langle \sin 2\theta_i \rangle = 0$, and therefore

$$r = \sqrt{\left(\frac{1}{N} \sum_{i=1}^{N} \cos 2\theta_i\right)^2} = \langle \cos 2\theta_i \rangle.$$  \hspace{1cm} (13)

Thus, $COOP = 2\langle \cos^2 \theta_i \rangle - 1 = \langle \cos 2\theta_i \rangle = r$.

**References**
