In our model, clusters grow over $T$ time steps and gravity selection then occurs. At the beginning of each time step, cluster division can occur. Following division, clusters grow, such that any cluster of size $x$ cells will be $G(x)$ cells by the end of the time step. Time point $t = 0$ marks the beginning of the first time step and time $t = T$ marks the point of gravity selection. Upon selection, a cluster of $x$ cells survives with probability $S(x)$, which we take to be a non-decreasing function. The maximal reproductive output for a cluster of size $x$ at time $t$ is given by $F(x, t)$. For us, this output function is simply a means to determine the optimal way for clusters to split, which can depend on both size and time. In our scheme, a cluster of $x$ cells can split into two clusters of sizes $p$ and $x-p$ (where $0 \leq p \leq x/2$). Because it is possible for the cluster not to split (i.e., if $p = 0$), we can simultaneously address the optimal rate of division along with optimal (a)symmetry.

A backwards recursion for maximal reproductive output can be formulated:

$$F(x, t) = \max_{0 \leq p \leq x/2} (F(G(p), t+1) + F(G(x-p), t+1))$$

Suppose that $G(x) = ax$ (where $a$ is an integer greater than unity) and $F(x, t+1)$ is concave. We note that $F(x, t+1)$ is only defined for integer values of $x$, so the standard requirement ($\frac{d^2F(x,t+1)}{dx^2} \leq 0$) is replaced by the following condition:

$$(F(x+1, t+1) - F(x, t+1)) - (F(x, t+1) - F(x-1, t+1)) \leq 0$$

If condition 2 holds for all values of $x$, then we have the following proposition.

**Proposition**

Given that $F(x, t+1)$ is concave by condition 2; for all integer values of $p$, where $0 \leq p \leq \frac{x}{2}$:

$$F(ap, t+1) + F(a(x-p), t+1) \leq \begin{cases} 2F\left(\frac{ax}{2}, t+1\right) & \text{if } x \text{ is even} \\ F\left(\frac{a(x+1)}{2}, t+1\right) + F\left(\frac{a(x-1)}{2}, t+1\right) & \text{if } x \text{ is odd} \end{cases}$$

**Proof**

Here we use a proof by induction. Consider the case where $x$ is even. Let $n = a\left(\frac{x}{2} - p\right)$ (for any defined value of $p$, $n$ will be some non-negative integer value). Condition 3 can be rewritten as:

$$F\left(\frac{ax}{2}, t+1\right) \geq \frac{F\left(\frac{ax}{2} - n, t+1\right) + F\left(\frac{ax}{2} + n, t+1\right)}{2}$$

Here, we will consider all non-negative integer values of $n$ (even those that don’t correspond to an integer value of $p$). Condition 4 clearly holds for $n = 0$. Additionally, it holds for $n = 1$ because condition 2 can be rewritten (with $x$ replaced by $\frac{x}{2}$) as

$$F\left(\frac{ax}{2}, t+1\right) \geq \frac{F\left(\frac{ax}{2} - 1, t+1\right) + F\left(\frac{ax}{2} + 1, t+1\right)}{2}$$

We now assume that condition 4 holds for $n$ and show that it must hold for $n + 1$. If $F(x, t+1)$ is a concave function, then we are guaranteed

$$F(x - 1, t + 1) \leq 2F(x, t + 1) - F(x + 1, t + 1)$$

$$F(x + 1, t + 1) \leq 2F(x, t + 1) - F(x - 1, t + 1)$$
Using conditions 6 and 7, the following holds:
\[ F \left( \frac{ax}{2} + (n + 1), t + 1 \right) + F \left( \frac{ax}{2} - (n + 1), t + 1 \right) \leq F \left( \frac{ax}{2} + n, t + 1 \right) + F \left( \frac{ax}{2} - n, t + 1 \right) \]

The following condition holds
\[ F \left( \frac{ax}{2} + n, t + 1 \right) + F \left( \frac{ax}{2} - n, t + 1 \right) \leq F \left( \frac{ax}{2} + (n - 1), t + 1 \right) + F \left( \frac{ax}{2} - (n - 1), t + 1 \right) \]

To show condition 9, we note that condition 7 ensures
\[ F \left( \frac{ax}{2} + n, t + 1 \right) - F \left( \frac{ax}{2} + (n - 1), t + 1 \right) + F \left( \frac{ax}{2} - n, t + 1 \right) - F \left( \frac{ax}{2} - (n - 1), t + 1 \right) \leq 0 \]

However condition 7 also ensures
\[ F \left( \frac{ax}{2} + (n - 1), t + 1 \right) - F \left( \frac{ax}{2} + (n - 2), t + 1 \right) + F \left( \frac{ax}{2} - n, t + 1 \right) - F \left( \frac{ax}{2} - (n - 1), t + 1 \right) \leq 0 \]

And the same substitution can be repeatedly applied, which yields
\[ F \left( \frac{ax}{2} + n, t + 1 \right) - F \left( \frac{ax}{2} + (n - 2), t + 1 \right) + F \left( \frac{ax}{2} - n, t + 1 \right) - F \left( \frac{ax}{2} - (n - 1), t + 1 \right) \leq 0 \]

Thus, condition 9 follows.

Condition 9 shows that condition 8 can be rewritten as
\[ F \left( \frac{ax}{2} + (n + 1), t + 1 \right) + F \left( \frac{ax}{2} - (n + 1), t + 1 \right) \leq F \left( \frac{ax}{2} + n, t + 1 \right) + F \left( \frac{ax}{2} - n, t + 1 \right) \]

Given that we are assuming that condition 4 holds for \( n \), it now follows
\[ F \left( \frac{ax}{2} + (n + 1), t + 1 \right) + F \left( \frac{ax}{2} - (n + 1), t + 1 \right) \leq F \left( \frac{ax}{2} + t + 1 \right) \]

Thus, condition 4 holds for \( n + 1 \). Thus, this condition will hold for all integer values of \( n \), which certainly ensures that it will hold for all integer values of \( p \) (where \( 0 \leq p \leq \frac{a}{2} \)). The case where \( x \) is odd follows a similar argument. This completes the proof. \( \square \)

Condition 4 is essentially an instance of Jensen’s inequality. Using Eq. 1 and condition 3,
\[ F(x, t) = \begin{cases} 2F \left( \frac{ax}{2} + t + 1 \right) & \text{if } x \text{ is even} \\ F \left( \frac{ax}{n+1} + t + 1 \right) + F \left( \frac{ax-1}{n+1} + t + 1 \right) & \text{if } x \text{ is odd} \end{cases} \]

Suppose that \( x \) is even; then Eq. 15 ensures
\[ (F(x, t) - F(x-1, t)) - (F(x-1, t) - F(x-2, t)) = 2F \left( \frac{ax}{2}, t + 1 \right) - 2F \left( \frac{ax}{2}, t + 1 \right) \]
\[-2F \left( \frac{ax-2}{2}, t + 1 \right) + 2F \left( \frac{ax-2}{2}, t + 1 \right) \]
\[= 0 \]
Suppose that $x$ is odd; then Eq. 15 ensures

\begin{equation}
(F(x, t) - F(x - 1, t)) - (F(x - 1, t) - F(x - 2, t)) = F\left(\frac{a(x + 1)}{2}, t + 1\right) + F\left(\frac{a(x - 1)}{2}, t + 1\right) - 2F\left(\frac{a(x - 1)}{2}, t + 1\right) - 2F\left(\frac{a(x - 1)}{2}, t + 1\right) + F\left(\frac{a(x - 1)}{2}, t + 1\right) + F\left(\frac{a(x - 3)}{2}, t + 1\right)
\end{equation}

\begin{equation}
= \left[ F\left(\frac{a(x + 1)}{2}, t + 1\right) - F\left(\frac{a(x - 1)}{2}, t + 1\right) \right] - \left[ F\left(\frac{a(x - 1)}{2}, t + 1\right) - F\left(\frac{a(x - 3)}{2}, t + 1\right) \right]
\end{equation}

Because $F(x, t + 1)$ is concave by assumption, this means that the right-hand side of Eq. 17 is less than or equal to zero; thus, for all relevant $x$

\begin{equation}
(F(x, t) - F(x - 1, t)) - (F(x - 1, t) - F(x - 2, t)) \leq 0
\end{equation}

This means that if $F(x, t + 1)$ is concave over integer values of $x$, then $F(x, t)$ will be as well. Thus, if $F(x, T) = S(x)$ is concave, then $F(x, t)$ will be concave for all values of $t$. This means that there will be no better strategy than splitting the group into two equal pieces (or as close as possible).

If it were possible for $F(x, t + 1)$ to be convex for all integer values ($(F(x + 1, t + 1) - F(x, t + 1)) - (F(x, t + 1) - F(x - 1, t + 1)) \geq 0$), then a similar argument would show that $F(x, t)$ will be convex as well. In such a case, it would be best for the cluster not to split at all ($p = 0$ is the optimal value). Given that $S(x)$ is a probability (and thus bounded at unity) this function cannot be convex for all values of $x$. Therefore, if $F(x, T) = S(x)$, then it will generally not be the case that $F(x, t)$ is convex for arbitrary values of $x$ and $t$. It is possible for $S(x)$ (and $F(x, t)$) to be convex for some values of $x$ and concave for other values. In such a case, it is possible for the optimal division strategy to depend on size and time. For instance, in Figure S1, the results of a program to calculate the optimal division values ($p$ as a function of $x$ and $t$) using Eq. 1 are given for a set of different functions for $F(x, T)$ (in the figure, we do impose a maximum size for a cluster). Thus, we see that there are circumstances where a cluster should not divide at certain sizes and divide evenly at other sizes.