Text S3

S3 Computational complexity with DiPDE

For a given time step $dt$ and number $N$ of LIF neurons to be simulated, the computational cost involved with a NEST simulation for a feed-forward network is $O(N/dt)$. The $1/dt$ dependence follows from the fact that the number of time steps involved in the simulation $N_t \sim 1/dt$. In our DiPDE simulations, there is no reference to a fixed number of neurons in the population; we solve for the probability distribution $p(v,t)$ for the neuronal membrane potential to be at some $v$ between 0 and $\theta$ at time $t$. The computational cost for a DiPDE simulation with time step $dt$ is dominated by the synaptic input step. As outlined in (Methods: Numerical Solutions), this step involves multiplying a $(N_v \times 2N_v)$ transition matrix $T$ with a $(N_v \times 1)$ vector of the probability distribution $p(v,t_k)$ for the membrane potential at time $t_k$. As in equation (15), $N_v = -\frac{\tau_m}{dt} \ln \left(1 - e^{-dt/\tau_m} \right)$ is the number of voltage bin-edges generated with our geometric binning scheme. Thus, the computational cost for a DiPDE simulation of a feed-forward network with external input rate $f$ determined by a homogeneous Poisson process scales asymptotically as $O(N_v^2/dt)$. For a $\delta$-function or bimodal distribution of synaptic weights, the transition matrix $T$ is sparse. In such cases, the computational cost scales asymptotically as $O(nnz/dt)$ where $nnz \sim N_v$ is the number of non-zero elements in the sparse transition matrix $T$.

Table (S1) shows the total simulation times using NEST (with different numbers of neurons $N$) and DiPDE for different choices of time step $dt$, for the feed-forward network of Fig. (1). Note that the transition matrix $T$ in this case is a $N_v \times 2N_v$ sparse matrix which keeps track of the effect of super-threshold synaptic inputs. Ignoring the effect of the excess synaptic input and re-setting the membrane potential to zero would lead to a $N_v \times N_v$ sparse matrix.

With an unconditionally stable numerical scheme to solve the Fokker-Planck equation, the computational cost involved would scale asymptotically as $O(N_v'/dt')$, where $N_v'$ is the number of uniform bins used to discretize the voltage between 0 and $\theta$ and $dt'$ is the time step used.