Figure S5: **Masking of nonlinearity by noise I.** Each panel shows an example of fitting a straight line (with weighted least square regression) to 15 averaged random trials of equation \((S10)\). In this figure \(\beta_1 = 1.73464\), and noise levels were set to match those in Figure 3a (shaded area). The purpose of this simulation is illustrate how noise can hide the nonlinearity of the m-Tau function. The statistical parameters associated with the weighted linear regression fit are indicated in the headline of each panel.

S2 Nonlinearity of the m-Tau Function

In this section we address whether \(\tau_{\text{mod}}\) can reproduce some of the reported properties of extracellular recordings from locusts’ DCMD. This section will focus on the experimental data from Gabbiani et al. [1]. We manually resampled their data points (plus standard deviations) from their Figure 4a on page 1128 with an ad hoc programmed graphical interface (resampled data are shown in Figure 3a & 4, respectively, and a screenshot of the graphical interface is shown in Figure S16a). Their data suggest a linear relationship between relative time of peak firing rate and the half-size to velocity ratio \(\kappa \equiv l/v:\)

\[
t_{\text{peak}}^{(n)} = \alpha \kappa + \delta
\]

where relative peak time will be denoted by \(t_{\text{peak}} \equiv t_c - t_{\text{max}}\). Notice that (i) \(t_{\text{max}}\) is the absolute time of the maximum, (ii) \(t_{\text{peak}}\) is always positive or zero. Gabbiani et al. [1] found slope \(\alpha = 4.7 \pm 0.3\) and
Figure S6: Masking of nonlinearity by noise II. Same as the last figure, but here \( \beta_1 = 1.52651 \), and the noise level in equation (S10) was set to \( \xi(\kappa) = 0.87\kappa \). The latter equation was proposed by Gabbiani et al. (equation 8, page 1129 in reference [1]). Figure panel d is identical with Figure 3b.

Intercept \( \delta = -27 \pm 3 \text{ ms} \) (±1 SD, \( N = 15 \) neurons). We used weighted linear regression for fitting their data and obtained \( \alpha = 4.61 \pm 0.179 \) and \( \delta = -30 \pm 1 \text{ ms} \), respectively (figure 3a).

Figure 4 illustrates how the relative peak time \( t_{\text{peak}} \) of the \( \tau_{\text{mod}} \)-maxima depends on \( \kappa \):

\[
t_{\text{peak}} = \sqrt{\kappa \left( \frac{2}{\beta_1 + \kappa} \right)} \quad (S6)
\]

In Figure 4, the values of \( \beta_1 \) were chosen according to

\[
\beta_1 = 2 \left( \frac{t_{\text{peak}}^2}{\kappa - \kappa} \right)^{-1} \quad (S7)
\]

with values for \((\kappa, t_{\text{peak}})\) taken at selected data points marked by red square symbols. Within the displayed domain of \( \kappa \) and range \( t_{\text{peak}} \), the curve \( t_{\text{peak}}(\kappa) \) appears almost linear for \( \beta_1 = 111.70 \), but is visibly bended for \( \beta_1 = 1.85 \). The slope of \( t_{\text{peak}}(\kappa) \) can be obtained by linearization of equation (S6) around some constant \( \kappa_0 \) (Taylor series up to the first order):

\[
t_{\text{peak}}(\kappa) \approx t_{\text{peak}}(\kappa_0) + (\kappa - \kappa_0) \cdot \left. \frac{\partial t_{\text{peak}}}{\partial \kappa} \right|_{\kappa=\kappa_0} \quad (S8)
\]

The last equation represents a straight line (tangent at \( \kappa_0 \)) with slope

\[
\frac{\partial t_{\text{peak}}}{\partial \kappa} = \kappa + \frac{1}{\beta_1} \quad \sqrt{\kappa \left( \frac{2}{\beta_1} + \kappa \right)} \quad (S9)
\]

which needs to be evaluated at \( \kappa = \kappa_0 \). The slope approaches 1 for large \( \beta_1 \), because the terms with \( 1/\beta_1 \approx 0 \). Because then the slope does not dependent on \( \kappa \) anymore, \( t_{\text{peak}} \) equals a straight line in the
Figure S7: Statistics of Line Fit I. The coefficient of determination ($R^2$) is the proportion of variance of the data which can be explained by the (here linear) model. The continuous line is the mean of the $n = 15$ data points at each of 81 values of $\beta_1$. The shaded area indicates one standard deviation. The broken line is the median value (0.92) across all data points.

Figure S8: Statistics of Line Fit II. See the legend of Figure 5 for computation details. Continuous lines indicate the mean of the $n = 15$ data points at each value of $\beta_1$. Shaded areas indicate one standard deviation, and the broken line is the median across all data points.

(a) Mean Square Error
(b) Kolmogorov-Smirnov Test

limit $\beta_1 \to \infty$.

If lines are fitted “brute-force” to the nonlinear curves $t_{\text{peak}}(\kappa)$ for finite values of $\beta_1$, then both their slopes $\alpha$ and their intercepts $\delta$ will increase with decreasing values of $\beta_1$ (see corresponding values in
Figure S9: Statistics of Line Fit III. Details are described in the legend of Figure 5. Continuous lines = mean of data points (n = 15) at each β₁ (81 values). Shaded areas = one standard deviation. Broken line = median across all data points. (a) Ratio of the model mean square to error mean square ("F-statistics", median=108). (b) Probability of obtaining observed F-values if a reduced model is used instead of the full linear model (median p value 2.37 ¥ 10⁻⁵). Note the logarithmic scaling of the ordinate.

Figure 4 written in small font size under the values of β₁. This hints at a positive correlation between α and δ, that was also reported by Gabbiani et al. [1].

The nonlinear dependence of t_peak on κ is nevertheless not in agreement with Gabbiani et al.’s findings [1], because they reported a linear relationship (equation S5). We observed, however, that sufficiently high noise levels can contribute to hiding the nonlinearity. Our point is illustrated with figures S5 and S6, where each figure panel represents a simulation in which Gaussian noise ξ with zero mean and with standard deviation σ̂ has been added to equation (S6). Accordingly, the simulation protocol is defined by (i) substitution of equation (S6) by

\[ t_{\text{peak}} = \sqrt{\kappa \left( \frac{2}{\beta_1 + \kappa} \right) + \sigma \xi} \]  

(S10)

(ii) averaging 15 results of the “noisified” version of t_peak, and (iii) fitting a line to the averaged (κ, t_peak) data. For the simulation we need to specify β₁ and the noise level σ̂ = σ(κ) in the last equation. For the noise level, two settings were considered. First, β₁ = 1.73464 ± 0.001516 (median ±σ̂rob) for the simulations shown in Figure S5, and we measured the standard deviation ̂σ at each κ directly from Figure 3a (shaded area). Second, β₁ = 1.52651 ± 0.002068 in Figure S6, and noise levels were assumed to be linearly increasing with κ, according to \( \sigma = 0.87\kappa \) (equation 8 on page 1129 in reference [1]).

The specific values of β₁ for the two noise levels were determined numerically with a “stochastic optimization algorithm”, which works as follows. First, a candidate value of β₁ is chosen, and N = 15 noisified instances of equation (S10) were averaged. In this way we obtained a mean t_peak as a function of κ, and associated standard deviations. The inverse of the squared standard deviations were used as weights for a weighted linear regression fit. From the fit a slope value was obtained, which was accepted if \( R^2 > 0.9, F > 50, \) and KS-test \( p > 0.5 \) were simultaneously fulfilled (acceptance with \( p = 0.25 \) for the first setting, and \( p = 0.67 \), respectively, for the second setting). Otherwise the slope value was ignored. All accepted slope values were subsequently averaged (typically 5000), and compared with the target slope α = 4.7 from Gabbiani et al. [1]. The value of β₁ was increased or decreased as a result of the comparison, and the algorithm continued to modify β₁ until a convergence criterion was met. The algorithm was run several times, and the median of the resulting values of β₁ was taken.

In all simulations shown in Figures S5 and S6, the (simulated) data points are consistent with linearity. The statistical parameters of each weighted regression result (as indicated at the top of each figure panel) are consistent with a linear model, too. Thus, our nonlinear model for t_peak (equation S6) can appear as being linear in the presence of suitable noise levels.

Gabbiani et al. noted a positive correlation between α and δ (correlation coefficient 0.76, cf. Figure 4b, page 1128 in reference [1]). Figure 4 suggests such a correlation for lines fitted to equation (S6). We studied this issue in more detail by the procedure described in the legend of Figure 5. For each of 81 values of β₁ ∈ [0.9, 1.8], n = 15 data points were generated. Each data point in turn represents inter-
cept $\delta$ and slope $\alpha$ of a line fit to $N = 15$ averaged random trials of the “noisified” equation (S6) (i.e., equation S10). Figure 5 shows a correlation between intercept and slope. The addition of noise decreases correlation somewhat, because the long main axis of the ellipse in Figure 5 has a smaller slope than the straight line that characterizes the noise-free situation.

The statistical parameters (Figures S7 to S9) for the $15 \times 81$ linear regressions provide further support to that the nonlinearity is successfully hidden by the noise. Specifically, the mean square error (MSE, Figure S8a) decreases with increasing $\beta_1$. This behavior is in line with equations (S8) and (S9), respectively, where we noted that $t_{\text{peak}}(\kappa)$ eventually approaches a straight line for increasing values of $\beta_1$.

If we ignored the standard deviations associated with the data points $t_{\text{peak}}(\kappa)$, we could use ordinary linear regression or a robust regression method to fit the data. In that case, we would obtain somewhat higher slope values for all fits. For example, ordinary linear regression of the data from Gabbiani et al. [1] would yield $\alpha = 5.19$ and $\delta = -40\ ms$. With $\beta_1 = 0.819273$, the corresponding linear regression fits of equation (S10) would again result in consistent predictions for the two considered noise settings. Thus, our argument that the m-Tau nonlinearity could be hidden by noise does not depend critically on the choice of the fitting method.
References