Figure S3

**Figure S3: Two-time-scale decomposition of the single-positive-loop system (1) in the main text.** (A)-(C) The zero-order approximation (dashed line) and the whole solution (solid line) of $c$ (red) and $b$ (black) in response to the signal $s(t) = 1 + \sin(2\pi \omega t)$. (A) High frequency, $\omega = 1$. (B) Medium frequency, $\omega = 0.1$. (C) Low frequency, $\omega = 0.01$. (D)-(F) The noise-free approximation (blue) versus zero-order approximation (red) of $c$ in response to the signal $s(t) = 1 + \sin(2\pi \omega t)$. (D) High frequency, $\omega = 1$. (E) Medium frequency, $\omega = 0.1$. (F) Low frequency, $\omega = 0.01$. (G) The slow quasi-periodic noise profile, $s(t) = 1 + (\sin(2\pi \omega t) + \sin(2\sqrt{2}\pi \omega t))/2, \omega = 0.01$. (H) The noise profile of $s(t) = 1 + \sum_{k=1}^{2000} \frac{\xi_k}{k} \sin(2\pi k \omega t), \xi_k \sim N(0, 1), \omega = 0.01$. (I) The corresponding output of (G). (J) The corresponding output of (H). All simulations use the same parameters as in Figure 3, unless otherwise specified. The initial condition is $(c, b) = (0.1, 0)$. 