**Text S3. Obtaining the logarithmic Hill equation**

If Bcd is the only factor regulating Hb production, we can take the fraction of *hb* promoter occupancy (*Fr*) as proportional to the normalized protein concentration, which is equivalent to the normalized signal intensity in immunostaining assays (H/H_{max}, Eq. S3.1), since immunofluorescent staining signal is proportional to protein concentration plus a nonspecific background, as already shown [1].

\[ Fr = \frac{H}{H_{max}} \]  

(S3.1)

The Hill equation can then be written as

\[ Fr = \frac{Bcd^n}{Bcd^n + Bcd_{1/2}^n} \]  

(S3.2)

where Bcd_{1/2} is the Bcd concentration necessary for half maximal *hb* promoter occupancy.

Through simple algebra, the equation above can be rewritten to

\[ Fr.Bcd^n + Fr.Bcd_{1/2}^n = Bcd^n \]  

(S3.3)
and further rearranged to

\[ Fr \cdot Bcd_{1/2}^n = Bcd^n (1 - Fr) \]  \hspace{1cm} (S3.4)

Dividing both sides by \( Bcd_{1/2}^n (1 - Fr) \) and taking logarithms gives

\[ n = \frac{\ln \left( \frac{Fr}{1 - Fr} \right)}{\ln \left( \frac{Bcd}{Bcd_{1/2}} \right)} \]  \hspace{1cm} (S3.5)

Equation (S3.5) was used in the Discussion regarding calculations using the Hill equation.

Reference