

Table S2. List of reactions and the rate equations for the model of tau pathophysiology.

| Reaction Type | Description | Kinetic Law | Reactants | Products | Params | Units | |
|--|---|--|---|------------------------|--------------------|--------------------------------------|--------------------------------------|
| Synthesis of 3R tau | Tau synthesis | $r(1) = k(1)$ | - | Tau0 | k(1) | $\mu\text{M}\cdot\text{s}^{-1}$ | |
| Phosphorylation & dephosphorylation of free 3R tau | Phosphorylation of newly synthesized tau | $r(2) = k(2)*x(1)*x(23)/(k(3)+x(1))$ | Tau0, ATP | TauN, ADP | k(2) | $\mu\text{M}^{-1}\cdot\text{s}^{-1}$ | |
| | Dephosphorylation of normally phosphorylated tau | $r(3) = k(4)*x(2)/(k(5)+x(2))$ | TauN | Tau0 | k(3) | μM | |
| | Phosphorylation of normally phosphorylated tau | $r(4) = k(6)*x(2)*x(23)/(k(7)+x(2))$ | TauN, ATP | TauH, ADP | k(4) | s^{-1} | |
| | Phosphorylation of normally phosphorylated tau | $r(5) = k(6)*x(2)*x(23)/(k(7)+x(2))$ | TauN, ATP | TauH, ADP | k(5) | μM | |
| | Dephosphorylation of abnormal/misfolded tau | $r(5) = k(8)*x(3)/(k(9)+x(3))$ | TauH | TauN | k(6) | $\mu\text{M}\cdot\text{s}^{-1}$ | |
| Microtubule binding and release of 3R tau species | Conformation change favoring MT binding | $r(6) = k(10)*x(1)$ | Tau0 | Tau0* | k(7) | μM | |
| | Conformational change back to original | $r(7) = k(11)*x(7)$ | Tau0* | Tau0 | k(8) | s^{-1} | |
| | Binding of newly synthesized tau to MT | $r(8) = k(12)*x(7)*x(10)$ | Tau0*, MT | Tau0*-MT | k(9) | $\mu\text{M}^{-1}\cdot\text{s}^{-1}$ | |
| | Release of newly synthesized tau from MT | $r(9) = k(13)*x(4)$ | Tau0*-MT | Tau0*, MT | k(10) | s^{-1} | |
| | Conformation change favoring MT binding | $r(10) = k(14)*x(2)$ | TauN | TauN* | k(11) | s^{-1} | |
| | Conformational change back to original | $r(11) = k(15)*x(8)$ | TauN* | TauN | k(12) | μM | |
| | Binding of normally phosphorylated tau to MT | $r(12) = k(16)*x(8)*x(10)$ | TauN*, MT | TauN*-MT | k(13) | $\mu\text{M}^{-1}\cdot\text{s}^{-1}$ | |
| | Release of normally phosphorylated tau from MT | $r(13) = k(17)*x(5)$ | TauN*-MT | TauN*, MT | k(14) | s^{-1} | |
| | Conformation change favoring MT binding | $r(14) = k(18)*x(3)$ | TauH | TauH* | k(15) | s^{-1} | |
| | Conformational change back to original | $r(15) = k(19)*x(9)$ | TauH* | TauH | k(16) | μM | |
| | Binding of misfolded tau to MT | $r(16) = k(20)*x(9)*x(10)$ | TauH*, MT | TauH*-MT | k(17) | $\mu\text{M}^{-1}\cdot\text{s}^{-1}$ | |
| | Release of misfolded tau from microtubules | $r(17) = k(21)*x(6)$ | TauH*-MT | TauH*, MT | k(18) | s^{-1} | |
| Free 3R tau degradation by non-ubiquitin dependent proteasome | Degradation of newly synthesized tau | $r(18) = k(22)*x(1)*x(21)*x(23)$ | Tau0, 20S, ATP | ADP, 20S | k(19) | $\mu\text{M}^{-2}\cdot\text{s}^{-1}$ | |
| | Degradation of phosphorylated tau | $r(19) = k(23)*x(2)*x(21)*x(23)$ | TauN, 20S, ATP | ADP, 20S | k(20) | $\mu\text{M}^{-2}\cdot\text{s}^{-1}$ | |
| | Degradation of misfolded tau | $r(20) = k(24)*x(3)*x(21)*x(23)$ | TauH, 20S, ATP | ADP, 20S | k(21) | $\mu\text{M}^{-2}\cdot\text{s}^{-1}$ | |
| Phosphorylation & dephosphorylation of microtubule-bound 3R tau | Phosphorylation of MT-bound newly synthesized tau | $r(21) = k(25)*x(4)*x(23)/(k(26)+x(4))$ | Tau0*-MT, ATP | TauN*-MT | k(22) | $\mu\text{M}^{-1}\cdot\text{s}^{-1}$ | |
| | Dephosphorylation of MT-bound normally phosphorylated tau | $r(22) = k(27)*x(5)/(k(28)+x(5))$ | TauN*-MT | Tau0*-MT | k(23) | μM | |
| | Phosphorylation of MT-bound normally phosphorylated tau | $r(23) = k(29)*x(5)*x(23)/(k(30)+x(5))$ | TauN*-MT, ATP | TauH-MT | k(24) | $\mu\text{M}^{-1}\cdot\text{s}^{-1}$ | |
| | Dephosphorylation of MT-bound abnormal/misfolded tau | $r(24) = k(31)*x(6)/(k(32)+x(6))$ | TauH-MT | TauN*-MT | k(25) | μM | |
| 3R Tau loading by Hsp70 | Association of misfolded tau with Hsp70 | $r(25) = k(33)*x(3)*x(11)$ | TauH, Hsp70 | TauH-Hsp70 | k(26) | $\mu\text{M}^{-1}\cdot\text{s}^{-1}$ | |
| | Hsp70 release | $r(26) = k(34)*x(15)$ | TauH-Hsp70 | TauH, Hsp70 | k(27) | s^{-1} | |
| 3R Tau Refolding | Exchange of Hsp70 for Hsp90 | $r(27) = k(35)*x(15)*x(12)$ | TauH-Hsp70, Hsp90 | TauH-Hsp90, Hsc70 | k(28) | $\mu\text{M}^{-1}\cdot\text{s}^{-1}$ | |
| | Restoration of TauH to Tau0 via Hsp90 | $r(28) = k(8)*x(16)/(k(9)+x(16))$ | TauH-Hsp90 | Tau0-Hsp90 | k(8) | s^{-1} | |
| Degradation of 3R tau by chaperones | Release of Tau0 from Hsp90 | $r(29) = k(36)*x(17)$ | Tau0-Hsp90 | Hsp90, Tau0 | k(9) | μM | |
| | Exchange of Hsc70 for CHIP | $r(30) = k(37)*x(15)*x(14)$ | TauH-Hsc70, CHIP | TauH-Hsc70-CHIP | k(30) | $\mu\text{M}^{-1}\cdot\text{s}^{-1}$ | |
| | Ubiquitination of tau | $r(31) = k(38)*x(18)$ | TauH-Hsc70-CHIP | TauHUB, Hsc70, CHIP | k(31) | s^{-1} | |
| | Binding of Bag-2 to the degradation complex | $r(32) = k(39)*x(18)*x(13)$ | TauH-Hsc70-CHIP, Bag2 | TauH-Hsc70-CHIP-Bag2 | k(32) | $\mu\text{M}^{-1}\cdot\text{s}^{-1}$ | |
| | Rescue from degradation by Bag2 | $r(33) = k(40)*x(19)$ | TauH-Hsc70-CHIP-Bag2 | TauH-Hsc70, CHIP, Bag2 | k(33) | s^{-1} | |
| Synthesis of 4R tau | Degradation of ubiquitinated tau | $r(34) = k(41)*x(20)*x(22)*x(23)$ | TauHUB, 26S, ATP | ADP, 26S | k(40) | $\mu\text{M}^{-2}\cdot\text{s}^{-1}$ | |
| | Synthesis of 4R tau | $r(35) = k(42)$ | - | Tau0 | k(42) | $\mu\text{M}\cdot\text{s}^{-1}$ | |
| | Phosphorylation & dephosphorylation of free 4R tau | Phosphorylation of newly synthesized tau | $r(36) = k(43)*x(25)*x(23)/(k(44)+x(25))$ | Tau0, ATP | TauN, ADP | k(43) | $\mu\text{M}^{-1}\cdot\text{s}^{-1}$ |
| | | Dephosphorylation of normally phosphorylated tau | $r(37) = k(45)*x(26)/(k(46)+x(26))$ | TauN | Tau0 | k(44) | μM |
| | | Phosphorylation of normally phosphorylated tau | $r(38) = k(47)*x(26)*x(23)/(k(48)+x(26))$ | TauN, ATP | TauH, ADP | k(45) | s^{-1} |
| | | Phosphorylation of normally phosphorylated tau | $r(38) = k(47)*x(26)*x(23)/(k(48)+x(26))$ | TauN, ATP | TauH, ADP | k(46) | $\mu\text{M}^{-1}\cdot\text{s}^{-1}$ |
| | | Dephosphorylation of abnormal/misfolded tau | $r(39) = k(49)*x(27)/(k(50)+x(27))$ | TauH | TauN | k(47) | μM |
| | Microtubule binding and release of 4R tau species | Conformation change favoring MT binding | $r(40) = k(51)*x(25)$ | Tau0 | Tau0* | k(48) | μM |
| | | Conformational change back to original | $r(41) = k(52)*x(31)$ | Tau0* | Tau0 | k(49) | s^{-1} |
| | | Binding of newly synthesized tau to MT | $r(42) = k(53)*x(31)*x(10)$ | Tau0*, MT | Tau0*-MT | k(50) | $\mu\text{M}^{-1}\cdot\text{s}^{-1}$ |
| Release of newly synthesized tau from MT | | $r(43) = k(54)*x(28)$ | Tau0*-MT | Tau0*, MT | k(51) | s^{-1} | |
| Conformation change favoring MT binding | | $r(44) = k(55)*x(26)$ | TauN | TauN* | k(52) | s^{-1} | |
| Conformational change back to original | | $r(45) = k(56)*x(32)$ | TauN* | TauN | k(53) | μM | |
| Binding of normally phosphorylated tau to MT | | $r(46) = k(57)*x(32)*x(10)$ | TauN*, MT | TauN*-MT | k(54) | $\mu\text{M}^{-1}\cdot\text{s}^{-1}$ | |
| Release of normally phosphorylated tau from MT | | $r(47) = k(58)*x(29)$ | TauN*-MT | TauN*, MT | k(55) | s^{-1} | |
| Conformation change favoring MT binding | | $r(48) = k(59)*x(27)$ | TauH | TauH* | k(56) | s^{-1} | |
| Conformational change back to original | | $r(49) = k(60)*x(33)$ | TauH* | TauH | k(57) | μM | |
| Binding of misfolded tau to MT | | $r(50) = k(61)*x(33)*x(10)$ | TauH*, MT | TauH*-MT | k(58) | $\mu\text{M}^{-1}\cdot\text{s}^{-1}$ | |
| Release of misfolded tau from microtubules | | $r(51) = k(62)*x(30)$ | TauH*-MT | TauH*, MT | k(59) | s^{-1} | |
| Free 4R tau degradation by non-ubiquitin dependent proteasome | Degradation of newly synthesized tau | $r(52) = k(63)*x(25)*x(23)*x(21)$ | Tau0, 20S, ATP | ADP, 20S | k(60) | $\mu\text{M}^{-2}\cdot\text{s}^{-1}$ | |
| | Degradation of phosphorylated tau | $r(53) = k(64)*x(26)*x(23)*x(21)$ | TauN, 20S, ATP | ADP, 20S | k(61) | $\mu\text{M}^{-2}\cdot\text{s}^{-1}$ | |
| | Degradation of misfolded tau | $r(54) = k(65)*x(27)*x(23)*x(21)$ | TauH, 20S, ATP | ADP, 20S | k(62) | $\mu\text{M}^{-2}\cdot\text{s}^{-1}$ | |
| Phosphorylation & dephosphorylation of microtubule-bound 4R tau | Phosphorylation of MT-bound newly synthesized tau | $r(55) = k(66)*x(28)*x(23)/(k(67)+x(28))$ | Tau0*-MT, ATP | TauN*-MT | k(63) | $\mu\text{M}^{-1}\cdot\text{s}^{-1}$ | |
| | Dephosphorylation of MT-bound normally phosphorylated tau | $r(56) = k(68)*x(29)/(k(69)+x(29))$ | TauN*-MT | Tau0*-MT | k(64) | μM | |
| | Phosphorylation of MT-bound normally phosphorylated tau | $r(57) = k(70)*x(29)*x(23)/(k(71)+x(29))$ | TauN*-MT, ATP | TauH-MT | k(65) | $\mu\text{M}^{-1}\cdot\text{s}^{-1}$ | |
| | Dephosphorylation of MT-bound abnormal/misfolded tau | $r(58) = k(72)*x(30)/(k(73)+x(30))$ | TauH-MT | TauN*-MT | k(66) | μM | |
| 4R Tau loading by Hsp70 | Association of misfolded tau with Hsp70 | $r(59) = k(74)*x(27)*x(11)$ | TauH, Hsp70 | TauH-Hsp70 | k(67) | $\mu\text{M}^{-1}\cdot\text{s}^{-1}$ | |
| | Hsp70 release | $r(60) = k(75)*x(34)$ | TauH-Hsp70 | TauH, Hsp70 | k(68) | s^{-1} | |
| | Exchange of Hsp70 for Hsp90 | $r(61) = k(76)*x(34)*x(12)$ | TauH-Hsp70, Hsp90 | TauH-Hsp90, Hsc70 | k(69) | $\mu\text{M}^{-1}\cdot\text{s}^{-1}$ | |
| 4R Tau Refolding | Restoration of TauH to Tau0 via Hsp90 | $r(62) = k(77)*x(35)/(k(9)+x(35))$ | TauH-Hsp90 | Tau0-Hsp90 | k(70) | s^{-1} | |
| | Release of Tau0 from Hsp90 | $r(63) = k(78)*x(36)$ | Tau0-Hsp90 | Hsp90, Tau0 | k(71) | μM | |
| Degradation of 4R tau by chaperones | Exchange of Hsc70 for CHIP | $r(64) = k(79)*x(34)*x(14)$ | TauH-Hsc70, CHIP | TauH-Hsc70-CHIP | k(72) | $\mu\text{M}^{-1}\cdot\text{s}^{-1}$ | |
| | Ubiquitination of tau | $r(65) = k(80)*x(37)$ | TauH-Hsc70-CHIP | TauHUB, Hsc70, CHIP | k(73) | s^{-1} | |
| | Binding of Bag-2 to the degradation complex | $r(66) = k(81)*x(37)*x(13)$ | TauH-Hsc70-CHIP, Bag2 | TauH-Hsc70-CHIP-Bag2 | k(74) | $\mu\text{M}^{-1}\cdot\text{s}^{-1}$ | |
| | Rescue from degradation by Bag2 | $r(67) = k(82)*x(38)$ | TauH-Hsc70-CHIP-Bag2 | TauH-Hsc70, CHIP, Bag2 | k(75) | s^{-1} | |
| | Degradation of ubiquitinated tau | $r(68) = k(83)*x(39)*x(22)*x(23)$ | TauHUB, 26S, ATP | ADP, 26S | k(76) | $\mu\text{M}^{-2}\cdot\text{s}^{-1}$ | |
| ATP generation and use by other processes | Generation of ATP | $r(69) = k(84)*x(24)$ | ADP | ATP | k(83) | s^{-1} | |
| | Use of ATP by other processes | $r(70) = k(85)*x(23)$ | ATP | ADP | k(84) | s^{-1} | |
| | Aggregation of 3R and 4R tau | Nucleation of Tau3 | $r(71) = k(86)*x(20)*2$ | TauH3RUB (2) | Nucleus3 | k(85) | $\mu\text{M}^{-1}\cdot\text{s}^{-1}$ |
| | | Dissociation of Tau3 Nucleus | $r(72) = k(87)*x(40)$ | Nucleus3 | TauH3RUB (2) | k(86) | s^{-1} |
| | | Nucleation of Tau4 | $r(73) = k(88)*x(39)*2$ | TauH4RUB (2) | Nucleus4 | k(87) | $\mu\text{M}^{-1}\cdot\text{s}^{-1}$ |
| | | Dissociation of Tau4 Nucleus | $r(74) = k(89)*x(41)$ | Nucleus4 | TauH4RUB (2) | k(88) | s^{-1} |
| | | Formation of Tau3, length 3 | $r(75) = k(90)*x(20)*x(40)$ | Nucleus3, TauH3RUB | Agg33 | k(89) | $\mu\text{M}^{-1}\cdot\text{s}^{-1}$ |
| | | Dissociation | $r(76) = k(91)*x(42)$ | Agg33 | Nucleus3, TauH3RUB | k(90) | s^{-1} |
| | | Formation of Tau3, 1 > 3 from Tau3 aggregate of length 3 | $r(77) = k(90)*x(20)*x(42)$ | TauH3RUB, Agg33 | Ap | k(91) | $\mu\text{M}^{-1}\cdot\text{s}^{-1}$ |
| | | Formation of agg of 1 > 3 from aggregates of 1 > 3 | $r(78) = k(90)*x(20)*x(43)$ | TauH3RUB, Ap | Ap | k(92) | $\mu\text{M}^{-1}\cdot\text{s}^{-1}$ |
| | | Dissociation of aggregates 1-3 | $r(79) = k(91)*x(43)$ | Ap | TauH3RUB, Ap | k(93) | s^{-1} |
| | | Formation of Tau4, length 3 | $r(80) = k(92)*x(39)*x(41)$ | Nucleus4, TauH4RUB | Agg43 | k(94) | $\mu\text{M}^{-1}\cdot\text{s}^{-1}$ |
| | | Dissociation | $r(81) = k(93)*x(44)$ | Agg43 | Nucleus4, TauH4RUB | k(95) | s^{-1} |
| | | Formation of Tau4, 1 > 3 from Tau3 aggregate of length 3 | $r(82) = k(92)*x(39)*x(44)$ | TauH4RUB, Agg43 | Bp | k(96) | $\mu\text{M}^{-1}\cdot\text{s}^{-1}$ |
| Formation of agg of 1 > 3 from aggregates of 1 > 3 | | $r(83) = k(92)*x(39)*x(45)$ | TauH4RUB, Bp | Bp | k(97) | $\mu\text{M}^{-1}\cdot\text{s}^{-1}$ | |
| Dissociation of aggregates 1-3 | | $r(84) = k(93)*x(45)$ | Bp | TauH4RUB, Bp | k(98) | s^{-1} | |