A Bayesian Approach to the Evolution of Metabolic Networks on a Phylogeny

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Abstract

The availability of genomes of many closely related bacteria with diverse metabolic capabilities offers the possibility of tracing metabolic evolution on a phylogeny relating the genomes to understand the evolutionary processes and constraints that affect the evolution of metabolic networks. Using simple (independent loss/gain of reactions) or complex (incorporating dependencies among reactions) stochastic models of metabolic evolution, it is possible to study how metabolic networks evolve over time. Here, we describe a model that takes the reaction neighborhood into account when modeling metabolic evolution. The model also allows estimation of the strength of the neighborhood effect during the course of evolution. We present Gibbs samplers for sampling networks at the internal node of a phylogeny and for estimating the parameters of evolution over a phylogeny without exploring the whole search space by iteratively sampling from the conditional distributions of the internal networks and parameters. The samplers are used to estimate the parameters of evolution of metabolic networks of bacteria in the genus Pseudomonas and to infer the metabolic networks of the ancestral pseudomonads. The results suggest that pathway maps that are conserved across the Pseudomonas phylogeny have a stronger neighborhood structure than those which have a variable distribution of reactions across the phylogeny, and that some Pseudomonas lineages are going through genome reduction resulting in the loss of a number of reactions from their metabolic networks.

Introduction

Biological networks are under continuous evolution and their evolution is one of the major areas of research today [1–6]. The evolution of biological networks can be studied using various approaches such as maximum likelihood and parsimony [7,8]. The maximum likelihood approach calculates the likelihood of evolution of one network into another by summing over all possible networks that can occur during the course of evolution under the given model. Parsimony, on the other hand, assumes minimum evolution and only considers those networks that correspond to the minimum number of changes between the two networks. However, the problem with these approaches is that enumeration of networks potentially occurring during evolution becomes impractical in the case of biological networks as the number of networks grows exponentially with the network size. Recently, the evolution of biological networks has been studied using stochastic approaches where efficient sampling techniques make the problem computationally tractable. For example, Wiuf et al. [5] used importance sampling to approximate the likelihood and estimate parameters for the growth of protein networks under a duplicate attachment model. Similarly, Ratmann et al. [6] used approximate Bayesian computation to summarize key features of protein networks. The authors also approximated the posterior distribution of the model parameters for network growth using a Markov Chain Monte Carlo algorithm.

In this work, we focus on metabolic networks. The evolution of metabolic networks is characterized by gain and loss of reactions (or enzymes) connecting two or more metabolites and can be described as a discrete space continuous time Markov process where at each step of the network evolution a reaction is either added or deleted until the desired network is obtained [9]. To give a biologically relevant picture of evolution some reactions may be defined as core (reactions that cannot be deleted during the course of evolution) or prohibited (reactions that cannot be added) in the given networks. The evolution of metabolic networks can then be studied using simple (independent loss/gain of reactions) or complex (incorporating dependencies among reactions) stochastic models of metabolic evolution. We previously presented a neighbor-dependent model for the insertion and deletion of edges from a network where the rates with which reactions are added or removed from a network depend on the fraction of neighboring reactions present in the network [9]. In this model, two reactions were considered to be neighbors if they shared at least one metabolite. The model is summarized in Section ‘Neighbor-dependent model’ below. The neighbor-dependent model depicts a biologically relevant picture of metabolic evolution by taking the network structure into account when calculating the rates of insertion and deletion of reactions from a network. The model is, however, limited in the sense that it does not allow one to measure the strength of the neighborhood structure affecting network evolution.
Metabolic networks correspond to one of the most complex cellular processes. Most organisms have a common set of reactions as a part of their metabolic networks that relate to essential processes such as generation of energy and the synthesis of important biological molecules, which are required for their survival. However, a large proportion of the reactions present in different organisms are specific to the needs of individual organisms. The regions of metabolic networks corresponding to these non-essential reactions are under continuous evolution. Using different models of evolution, we can ask important biological questions about the ways in which the metabolic networks of different organisms enable them to be well-adapted to the environments in which they live, and how these metabolic adaptations have evolved. We use a stochastic approach to study the evolution of metabolic networks and show that evolutionary inferences can be made using the structure of these networks. Our results indicate that plant pathogenic *Pseudomonas* are going through genome reduction resulting in the loss of metabolic functionalities. We also show the potential of stochastic approaches to infer the networks present at ancestral levels of a given phylogeny compared to deterministic methods such as parsimony.

Here, we present an extended model called the hybrid model that combines an independent edge model, where edges are gained or lost independently, and a neighbor-dependent model of network evolution [9] such that the rate of going from one network to another is a sum of the rates under the two models based on a parameter, which measures the probability of being in the neighbor dependent model. This allows estimation of the neighborhood effect during metabolic evolution. When modeling network evolution, we represent metabolic networks as directed hypergraphs [9–11], where an edge called a hyperedge represents a reaction and may connect any number of vertices or metabolites. Representing metabolic networks as hypergraphs not only captures the relationship between multiple metabolites involved in a reaction but also provides an intuitive approach to study evolution since loss or gain of reactions can be regarded as loss or gain of hyperedges.

We use the hybrid model to study the evolution of a set of metabolic networks connected over a phylogeny. Previous attempts to study the evolution of metabolic networks in a phylogenetic context include Dandekar et al. [12] and Peregrin et al. [13]. However, to our knowledge, the stochastic treatment of metabolic evolution over a phylogeny is an unexplored area. Here, the phylogenetic relationship between the networks is established using sequence data since the metabolic annotations available for the majority of genome-sequenced organisms are generated using automated annotation tools based on the similarity of predicted genes to genes of known function and, therefore, contain a huge amount of noise. In addition, we treat the branch lengths obtained using the sequence data as certain. The advantage of fixing branch lengths is that the calculations do not require summing over all branch lengths for the given tree. Calculating the likelihood over a phylogeny then requires a sum, over all possible networks that may have existed at the interior nodes of the tree, of the probabilities of each scenario of events. This is similar to the idea introduced by Felsenstein [14] for observing DNA sequences over a phylogeny. To sample the networks at internal nodes of the tree a Gibbs sampler [15,16] is presented that samples a network conditioned on its three neighbors, including a parent and two children networks, for given parameter values. A Gibbs sampler for estimating the parameters of evolution that encases the Gibbs sampler for internal networks sampling is also presented. The sampler estimates the evolution parameters without exploring the whole search space by iteratively sampling from the conditional distributions of the trees and parameters. We demonstrate the Gibbs sampler by estimating and comparing the evolution parameters for the metabolic networks of bacteria belonging to the genus *Pseudomonas*. The Gibbs sampler can also be used to infer the ancestral networks of a given phylogeny. This is shown by inferring the metabolic networks of *Pseudomonas* spp. ancestors.

### Methods

#### Neighbor-dependent model

In the neighbor-dependent model for the evolution of metabolic networks [9] hyperedges are inserted or deleted from a network depending on the fraction of neighboring hyperedges present in the network. Two hyperedges are considered as neighbors if they share a node. The model assumes that the number of nodes in a network remains fixed and there is a set $E$ such that $|E|=M$ of hyperedges connecting these nodes. The model also assumes the existence of a network called Reference Network which contains all these hyperedges. If the hyperedges in the reference network are labeled 1 to $M$ then any given network $x$ can be represented as a sequence of 0s and 1s such that the $i$-th entry $(0 < i \leq M)$ in the sequence is 1 if and only if the hyperedge labeled $i$ is present in the network $x$, and 0 otherwise. Let the rate matrix describing the evolution under the neighbor-dependent model be denoted by $\Gamma$. An entry $\gamma(x_i; x_j)$ in this rate matrix corresponds to the rate of going from a network $x$ to a network $x'$, which differs from $x$ at position $i$. In the neighbor-dependent model, the rate $\gamma(x_i'; x_j)$ of going from $x$ to $x'$ depends on $x_i$, $x_j$, and the neighboring hyperedges $\Psi(x_i)$ present in the network $x$, and is given as follows:

$$\gamma(x_i; x_j) = q(x_i, x_j)F(x_i, \Psi(x_j))$$

(1)

where the function $F$ corresponds to the neighborhood component and $q(x_i, x_j)$ is the appropriate entry from the $2 \times 2$ rate matrix $Q_i$ for the hyperedge $i$. The rate matrix $Q_i$ is given as

$$Q_i = \begin{bmatrix} -\lambda & \lambda \\ \mu & -\mu \end{bmatrix}$$

(2)

where $\lambda$ is the insertion rate and $\mu$ is the deletion rate. The neighborhood component $F(x_i, \Psi(x_j))$ weights the insertion and deletion rates by the proportion of neighbors present in the network and is given as follows:

$$F(x_i, \Psi(x_j)) = \begin{cases} \frac{1}{M+1}, & |\Psi(x_j)| > 0 \\ \frac{|\Psi(x_j)|}{\sum_{j \neq i} |\Psi(x_j)|}, & |\Psi(x_j)| = 0 \end{cases}$$

(3)

The denominator $\sum_{j \neq i} |x_j|$ in Equation 3 gives the number of hyperedges present in the current network.

#### Hybrid model of network evolution

Although the neighbor-dependent model summarized above produces a biologically relevant behavior whereby highly connected reactions are toggled more frequently than the poorly connected counterparts, it does not allow one to determine the
strength of the neighborhood structure effecting the evolution of metabolic networks. To overcome this limitation, a parameter can be introduced in the model that corresponds to the neighborhood effect during the course of metabolic network evolution.

Consider two networks \( x \) and \( x' \) which differ at position \( x_i \). The hybrid model combines the independent edge model where edges are added or deleted independently, and the neighbor-dependent model summarized above such that the rate of going from \( x \) to \( x' \) is the sum of the rates under the two models based on a parameter \( \delta \), \( 0 \leq \delta \leq 1 \), which specifies the probability of being in the neighbor-dependent model. The rate from \( x \) to \( x' \) is given as

\[
\omega(x'; x) = \delta \gamma(x'; x) + (1 - \delta) q(x, x')
\]

where the term \( \gamma(x'; x) \) is the rate under the neighbor-dependent model given by Equation 1 and the term \( q(x, x') \) is the rate under the independent edge model corresponding to the appropriate entry from the rate matrix \( Q \) given by Equation 2. Substituting the value of \( \gamma(x'; x) \) from Equation 1, the above equation can be simplified as follows.

\[
\omega(x'; x) = \delta q(x, x') F(x, x') + (1 - \delta) q(x, x')
\]

where the term \( F(x, x') \) corresponds to the neighborhood component given by Equation 3.

It can been seen from (4) that the model behaves under the independent edge model when \( \delta \) equals 0 and under the neighbor-dependent model described in the previous section when \( \delta \) equals 1. For example, consider the toy network \( H_1 \) shown in Figure 1A. The reference network \( H \) containing all allowed hyperedges for this example system is also shown in the figure. The system behavior for different values of \( \delta \) is illustrated in Figure S1 for the toy network \( H_1 \) when simulated under the hybrid model along with the number of neighbors for each hyperedge. The rates were calculated at each step using (4). An edge was then selected based on these rates and was inserted if absent from the current network and deleted otherwise. As expected, hyperedges evolve independently when \( \delta = 0 \), resulting in similar insertion frequencies for all hyperedges and increasingly reflecting their neighborhood as the value of \( \delta \) goes up to unity. The fitness of the model is discussed in the Section ‘Fitness of the hybrid model’ below.

**Evolution on a phylogeny**

Biological networks are connected over a phylogenetic tree which is known through sequence analysis. Calculating the likelihood over a phylogeny requires a sum, over all possible networks that may have existed at the interior nodes of the tree, of the probabilities of each scenario of events. For example, Figure 1A shows an example system containing three networks \( H_1, H_2 \) and \( H_3 \) with a phylogeny connecting the three networks shown in Figure 1B. Let the phylogenetic tree be denoted by \( T \). The likelihood of the tree \( T \) is given as follows.

\[
L_\Theta(T) = \sum_{H_{1,2,3}} \{ P_\Theta(H_{1,2,3}) P_{\Theta,01}(H_3|H_{1,2,3}) \}
\]

where \( \Theta \) denotes the parameters of the model, which is \( (\lambda, \mu) \) in the case of the neighbor-dependent model and \( (\lambda, \mu, \delta) \) in the case of the hybrid model. \( P_\Theta(H_{1,2,3}) \) is the marginal probability of observing the root and \( P_{\Theta,01}(H_3|H_{1,2}) \) denotes the pairwise likelihood of evolving from the network \( H_1 \) to the network \( H_3 \) conditioned on \( H_2 \) in time \( t \) for the given parameters.

In general, the likelihood of a tree with more than three networks can be calculated using the recursion described by Felsenstein [17]. The likelihood at an internal node \( N \) of the tree is given by the following recurrence relation

\[
L_\Theta(N) = \sum_{N_l} P_{\Theta,01}(N_l|N) L_\Theta(N_l) \sum_{N_r} P_{\Theta,01}(N_r|N) L_\Theta(N_r)
\]

where \( N_l \) and \( N_r \) are left and right descendants of the node \( N \). The likelihood of the complete tree \( T \) is then given as

\[
L_\Theta(T) = \sum_{N_{root}} P_{\Theta}(N_{root}) L_\Theta(N_{root})
\]

where \( P_{\Theta}(N_{root}) \) is the marginal probability of observing the root and \( L_\Theta(N_{root}) \) is given by Equation 6.

Evaluating Equations 5 and 7 requires an algorithm to systematically and efficiently sample networks at the internal nodes of a tree and a method to calculate the pairwise likelihood of network evolution. A Metropolis-Hastings algorithm to calculate the pairwise likelihood based on sampling paths between network pairs was described by Mithani et al. [9], which calculates the likelihood by summing over paths between the given network pairs. To sample networks at the internal node of a tree, a Markov chain can be constructed where states correspond to networks at the internal nodes. The networks can then be sampled using a Gibbs sampler [15,16] as described in the next section.

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**Figure 1. Toy networks connected by a phylogeny.** (A) Toy networks consisting of 13 nodes. The nodes are labeled from A to M (blue) and the hyperedges are labeled from 1 to 10 (red). The reference network consists of all allowed hyperedges for this example system. Networks \( H_1, H_2 \) and \( H_3 \) consist of subsets of the hyperedges from the reference network. (B) A phylogeny connecting the networks \( H_1, H_2 \) and \( H_3 \). doi:10.1371/journal.pcbi.1000868.g001
Sampling internal nodes

Given a set of networks related by a phylogenetic tree, the networks at the internal nodes of the tree can be sampled using a Gibbs sampler. The general idea is to sample each internal network by conditioning on its three neighbors (one parent and two children). This approach for sampling internal networks is similar to the one used by Holmes and Bruno [18] for DNA sequence alignment. However, instead of using linear sequences, the sampler takes into account the network structure when calculating the new state. The procedure is described below.

Consider a network \( X \) with its three neighbors \( Y_k \) with branch lengths \( t_k \), \( k = 1 \ldots 3 \). The new network \( X' \) is selected as follows.

1. For each hyperedge \( i \), calculate the \( 2 \times 2 \) rate matrix

\[
\Omega_X = Q_X [\delta F_X (i,Y(i)) + (1-\delta)]
\]

where \( \delta \) is the neighbor-dependence probability, \( Q \) is the rate matrix given by Equation 2 and the function \( F \) corresponds to the neighborhood component given by Equation 3.

2. Calculate, for each neighbor \( Y_k (k = 1, \ldots, 3) \), the transition probabilities \( P_{\Theta,i} (Y_k(1)|X(i)) = \exp(t_k \Omega_X) \).

3. Sample the new state \( s'_i \in \{0,1\} \) for hyperedge \( i \) from the distribution

\[
P(s'_i) \propto \pi(s_i) \prod_{k=1}^{3} P_{\Theta,i} (Y_k(i)|s_i)
\]

where \( \pi \) is the vector equilibrium probabilities and can be obtained by solving the equation \( \pi \Omega_X = 0 \).

Example Consider the network \( H_{1,2} \) in Figure 2 for which new state is to be calculated. Denote the network by \( X \). The three neighboring networks of the network \( H_{1,2} \) are the networks \( H_1, H_2 \) and \( H_{1,2,3} \) labeled as \( Y_1, Y_2 \) and \( Y_3 \) respectively. If \( f_l \) denotes the neighborhood component for hyperedge \( i \) then for the rate parameters \( \lambda \) (insertion) and \( \mu \) (deletion), and the neighbor-dependence probability \( \delta \) the rate matrix \( \Omega_X \) is written as

\[
\Omega_X = \begin{bmatrix}
-\lambda & \lambda \\
\mu & -\mu
\end{bmatrix} (\delta f_i + (1-\delta)).
\]

For simplicity, assume that \( \delta = 1 \). The system then behaves under the neighbor-dependent model and the rate matrix simplifies to

\[
\Omega_X = \begin{bmatrix}
-\lambda & \lambda \\
\mu & -\mu
\end{bmatrix} f_i
\]

The transition probability matrix of transforming \( X(i) \) to \( Y_i(i) \) is then given as

\[
P_{\Theta,i} (Y_i(i)|X(i)) = \exp(t_i \Gamma_X),
\]

\[
= \frac{1}{\lambda + \mu} \begin{bmatrix}
\lambda + \mu \exp(-t_i (\lambda + \mu)) & \lambda (1- \exp(-t_i (\lambda + \mu))) \\
\mu (1- \exp(-t_i (\lambda + \mu))) & \lambda + \mu \exp(-t_i (\lambda + \mu))
\end{bmatrix}
\]

The transition probability matrices \( P_{\Theta,i} (Y_2(i)|X(i)) \) and \( P_{\Theta,i} (Y_3(i)|X(i)) \) can be calculated in the similar fashion.

Once the transition probability matrices have been obtained, the sample for the new network \( X' \) can be drawn using Equation 8. For example, if the current configuration of the networks are taken as shown in Figure 2, then the sample for the new state \( s_1 \), for hyperedge 1 is drawn from the following distribution:

\[
P(s_1) \propto \pi(s_1) \prod_{k=1}^{3} P_{\Theta,i} (Y_k(1)|s_i).
\]

\[
= \pi(s_1) P_{\Theta,i} (1|s_1) P_{\Theta,i} (1|s_1) P_{\Theta,i} (0|s_1).
\]

The samples for hyperedges labeled 2 to 10 can be drawn in a similar fashion to obtain the new network.

Estimation of parameters

The Gibbs sampler described above samples the internal networks on a phylogenetic tree for given parameter values. This can be extended to estimate the parameters \( \Theta \) of evolution where \( \Theta \) equals \( (\lambda, \mu) \) in case of the neighbor-dependent model and \( (\lambda, \mu, \delta) \) in case of the hybrid model. One way is to nest it within another Gibbs Sampler which iteratively samples internal networks and parameters from the distributions \( P(T|\Theta) \) and \( P(\Theta|T) \) respectively. The general outline of the Gibbs sampler is as follows:

- Choose initial values for the parameters \( \Theta^{(0)} \).
- Generate \( T^{(0)} \) by using the procedure described in Section ‘Sampling internal nodes’ using \( \Theta^{(0)} \).
- Use \( T^{(0)} \) to generate \( \Theta^{(1)} \) by drawing from the distribution \( P(\Theta|T) \).
- Repeat \( n \) times to get subset of points \( (T^{(0)}, \Theta^{(0)}) \), where \( 1 \leq i \leq n \), are the simulated estimates from the joint distribution \( P(T, \Theta) \).

The samples for parameters can be drawn using a Metropolis-Hastings algorithm [19,20] as described next. Since the Metropolis-Hastings algorithm is a well-established method, it suffices here to give details about how a proposal for new parameters can
be generated. Readers interested in the general details of the algorithm are referred to Chapter 1 of Gilks et al. [21]. The performance of the Gibbs sampler is discussed in Text S1.

Parameter proposal

**Rates proposal.** For a given tree $T$, a proposal for the rate parameters can be generated from a gamma distribution

$$\gamma \sim \Gamma(k, \theta); \quad \gamma = \lambda, \mu$$

where $k$ is the shape parameter and $\theta$ is the scale parameter. The hyper-parameters $k$ and $\theta$ can be calculated from the given tree as described next.

Starting from root, calculate the proportion of insertion events $I_{H_{1,0} \rightarrow H_1}$ and the proportion of deletion events $D_{H_{1,0} \rightarrow H_1}$ as the average number of neighbors present in all the networks present at the leaves of the phylogeny. For example, if the network $x(j)$ is a leaf network, i.e. it occurs at the tip of the given phylogenetic tree, then calculate

$$N_{x(j)} = \frac{1}{M} \sum_{i \in \text{leaves}(T)} |\Psi(x_i(j))|$$

The parameter $x$ is then given as the mean of the average number of neighbors present in all the networks present at the leaves of the given phylogenetic tree. For a tree $T$ with $l$ leaves, this can be written as follows.

$$x = \frac{1}{l} \sum_{j \in \text{leaves}(T)} N_{x(j)}$$

The shape parameter $\beta$ corresponds to the average number of neighbors in the reference network (REF) and is given as

$$\beta = \frac{1}{M} \sum_{i \in \text{leaves}(T)} |\Psi(x_i(REF))|,$$

such that

**Proposal probability**

**Rates proposal.** The proposal probability $q(\hat{\lambda}, \hat{\mu} | \lambda, \mu)$ for the rate parameters is given as

$$q(\hat{\lambda}, \hat{\mu} | \lambda, \mu) = \prod_{\gamma \sim \hat{\lambda}} q(\gamma | \hat{\mu})$$

where

$$k_2 = 0.3 + 1 = 1.3$$

and

$$\theta_2 = 1.0.$$
where \( k \) and \( \theta \) are the hyper-parameters of the gamma distribution given by Equations 9 and 10 respectively when \( \gamma = \lambda \) and by Equations 11 and 12 otherwise.

**Dependence probability proposal.** The proposal probability \( q(\delta|\delta) \) for the dependence probability parameters is given as

\[
q(\delta|\delta) \propto \delta(1 - \delta)^\beta
\]

where \( z \) and \( \beta \) are the shape parameters of the beta distribution calculated using Equations 13 and 14 respectively.

**Pairwise likelihood.**

The Metropolis-Hastings procedure described above to sample parameters requires the likelihood of the tree when moving in the parameter space. The likelihood can be calculated using Equation 5 which in turn requires calculation of the pairwise likelihood between network pairs. The pairwise likelihood can be calculated using the Metropolis-Hastings algorithm described in Mithani et al. [9] which calculates the likelihood by summing over all paths between the given network pair. However, for the Gibbs sampler described above in Section ‘Estimation of parameters’ this seems impractical since it will require running the Metropolis-Hastings sampler for all network pairs. An alternate way is to use a pseudo-likelihood value when calculating the acceptance probability for parameters. We calculate the pseudo-likelihood for a given network pair by dividing the network into smaller sub-networks and multiplying the pairwise likelihoods of the individual sub-networks.

Let \( P_{\theta_i}(H_i|H_t) \) denote the pseudo-likelihood from the network \( H_t \) to the network \( H_i \) in time \( t \) for the given parameter values. This is given as

\[
P_{\theta_i}(H_i|H_t) = \prod_k P_{\theta_i}(H^k_i|H^k_t)
\]

where \( P_{\theta_i}(H^k_i|H^k_t) \) is the pairwise likelihood of evolving sub-network \( H^k_i \) into \( H^k_t \) calculated by solving the exponential \( \exp(tQ) \). The procedure to obtain sub-networks \( H^k_i \) containing at most \( N \) hyperedges is outlined below.

1. Initialize \( k = 1 \).
2. Select the hyperedge \( e \) with highest number of neighbors and add it to the network \( H^k_t \).
3. Add the top \( K \) neighbors of hyperedge \( e \) based on the number of neighbors to the network \( H^k_t \) where \( K = \min(N - 1, N - |H^k_t|) \).
4. Remove the hyperedges present in \( H^k_t \) from \( H_t \) and calculate the number of neighbors based on the remaining hyperedges in the network.
5. Repeat steps 2–4 until \( |H^k_t| < N \).
6. Increment \( k \) and repeat steps 2–5 until the network \( H_t \) is exhausted.

An example is given in Figure S2, which shows the sub-networks for the toy network \( H_1 \) shown in Figure 1 for different values of \( N \). The above procedure was used to calculate the pseudo-likelihood of evolution of the toy network \( H_1 \) to the network \( H_2 \) (Figure 1A) for different subnetwork sizes, and the results were compared against the likelihood obtained by the MCMC approach described in Mithani et al. [9] and the true likelihood values obtained by evaluating \( \exp(tQ) \). All likelihood values were conditioned on the starting network. The average CPU time taken by different approaches is shown in Figure 3 and the pseudo-likelihood values are listed in Table S1. The sub-network approach provides a reasonable approximation of the likelihood with a significant time advantage over the MCMC approach.

**Results**

**Fitness of the hybrid model.**

To see if the hybrid model fitted the metabolic network data better than the neighbor-dependent model, a likelihood ratio test was performed using the metabolic data for the bacteria belonging to the genus *Pseudomonas*. The results show that the hybrid model fits the metabolic data better than the neighbor-dependent model. For example, consider the metabolic networks in *Pseudomonas fluorescens* Pf0-1. The maximum likelihood estimates (MLEs) for the evolution of glycolysis/gluconeogenesis map [22] from *Pseudomonas fluorescens* Pf-5 to *P. fluorescens* Pf0-1 obtained using the Gibbs sampler described by Mithani et al. [9] were \((\lambda, \mu, \delta) = (2.6177, 0.4229)\) under the neighbor-dependent model and \((\lambda, \mu, \delta) = (0.4989, 0.1598, 0.2152)\) under the hybrid model. Using the MLEs, the likelihood of observing the data under each model was calculated. Assuming that evolution has been taking place for a long time, it is reasonable to use the equilibrium probability of a network to approximate the probability of observing the network. The equilibrium probabilities were calculated using the procedure described by Mithani et al. [9]. The maximum log likelihood obtained under the neighbor-dependent model equaled \(-76.53\) whereas the maximum log likelihood obtained under the hybrid model equaled \(-63.47\). The likelihood ratio test statistic \( \chi^2 \) was calculated as \( 2(76.53 - 63.47) = 26.13 \) under 3 degrees of freedom. The p-value \( P(\chi^2 > X^2) = 3.19 \times 10^{-7} \) on 1 degree of freedom suggests that the hybrid model fits the data better than the neighbor-dependent model.
neighbor-dependent model. The MLEs, maximum log-likelihoods and the p-values for different pathway maps in P. fluorescens PF0-1 used in this analysis are listed in Table 1. The low p-values for all the pathway maps suggest a better fit for the hybrid model compared to the neighbor-dependent model. Likelihood ratio tests for other genome-sequenced Pseudomonas strains used in this analysis showed similar results (data not shown).

The fit of the data was further tested by comparing the degree distributions of the nodes obtained by simulating network evolution under the neighbor-dependent and hybrid models. The MLEs for the evolution of networks obtained under the two models were used as the simulation parameters. For example, when evolving the pathway maps in P. fluorescens PF0-1, the parameter values listed in Table 1 were used. A total of 60,000 iterations were run with the first 10,000 iterations regarded as burn-in period. Samples were collected every 10th iteration and degree distributions were calculated. The results for the six pathway maps used in this analysis are shown in Figure 4 for P. fluorescens PF0-1 as an example which suggest a better fit for the hybrid model than the neighbor-dependent model. Similar results (data not shown) were obtained for the other genome sequenced Pseudomonas strains used in this analysis.

Toy networks
To test the Gibbs sampler described in Section ‘Sampling internal nodes’, the three network phylogeny shown in Figure 1 was used. The networks were sampled at the internal nodes for different rate combinations with the neighbor-dependence probability d kept constant at 1. The likelihood value was then calculated using Equation 5 by summing over the networks visited by the sampler at each internal node for each rate combination. When calculating the likelihood over the phylogeny, the pairwise likelihood was calculated using matrix exponentiation. A total of 25,000 iterations were run for each rate combination with the first 10,000 iterations regarded as burn-in period. The exact likelihood of the phylogeny was also calculated by matrix exponentiation using all $1024 = 2^{10}$ networks at each internal node. The likelihood values estimated using the networks visited by the Gibbs sampler were comparable to those obtained by summing over all 1024 networks. The true and estimated likelihood surfaces for a range of parameter values are shown in Figure S3.

We also ran the Gibbs sampler for parameter estimation for the toy networks. The sampler was run from a random starting value for 60,000 iterations with the first 10,000 iterations regarded as the burn-in period. The samples were collected every 10th iteration to reduce computational overhead relating to storage as well as the correlation between samples. A sample MCMC trace for the first 1,000 iterations of the sampler for the rate parameters is shown in Figure S4. The autocorrelation of parameters is plotted in Figure 5 suggesting an exponential decrease in the correlation as the lag between the samples increases. To test the performance of the sampler, the likelihood of evolution for different rate combinations visited by the sampler was also calculated using Equation 5 by summing over networks visited by the sampler with $d = 1$. As before, the pairwise likelihood was evaluated by calculating the exponential of the rate matrix. The maximum likelihood averaged over three runs was found to be $3.6583 \times 10^{-8} \pm 1.0307 \times 10^{-12}$ for parameters $(\lambda, \mu) = (0.8259, 0.4066)$ which is very close to the true likelihood obtained by matrix exponentiation (Figure S3).

### Table 1. Likelihood ratio test between the neighbor-dependent and hybrid models of metabolic evolution.

<table>
<thead>
<tr>
<th>Pathway map</th>
<th>Neighbor-dependent model</th>
<th>Hybrid model</th>
<th>LH ratio</th>
<th>$P(\chi^2 \geq X^2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Glycolysis/Gluconeogenesis</td>
<td>(2.6177, 0.4229)</td>
<td>(0.4989, 0.1598, 0.2152)</td>
<td>-63.47</td>
<td>26.13</td>
</tr>
<tr>
<td>Pentose phosphate pathway</td>
<td>(0.5680, 0.7144)</td>
<td>(0.0762, 0.2953, 0.4259)</td>
<td>-53.42</td>
<td>13.41</td>
</tr>
<tr>
<td>Lysine degradation</td>
<td>(0.0127, 1.0780)</td>
<td>(0.0063, 0.2926, 0.0159)</td>
<td>-52.40</td>
<td>14.05</td>
</tr>
<tr>
<td>Histidine metabolism</td>
<td>(0.7669, 0.3895)</td>
<td>(0.1852, 0.1643, 0.1370)</td>
<td>-47.28</td>
<td>13.89</td>
</tr>
<tr>
<td>Phenylalanine metabolism</td>
<td>(1.1035, 0.6856)</td>
<td>(1.0299, 1.0297, 0.0038)</td>
<td>-49.91</td>
<td>24.97</td>
</tr>
<tr>
<td>Pyruvate metabolism</td>
<td>(0.1648, 0.5065)</td>
<td>(0.0897, 0.1913, 0.1194)</td>
<td>-81.74</td>
<td>24.97</td>
</tr>
</tbody>
</table>

The maximum likelihood estimates (MLEs) of the parameter values ($\lambda$: insertion rate, $\mu$: deletion rate and $d$: neighbor dependence probability), maximum log-likelihoods, likelihood (LH) ratios, and the p-values for different pathway maps in P. fluorescens PF0-1 used in this analysis. The MLEs were obtained using the Gibbs sampler described by Mithani et al. [9] by evolving the networks from P. fluorescens PF-5 to P. fluorescens PF0-1. The equilibrium probability of a network was used as the likelihood of observing the network. The low p-values for all the pathway maps suggest a better fit for the hybrid model compared to the neighbor-dependent model. doi:10.1371/journal.pcbi.1000868.t001

Parameter estimation for metabolic networks in Pseudomonas
To study the metabolic evolution in bacteria, we used the Gibbs sampler to estimate the evolution parameters for the metabolic networks of bacteria belonging to the genus Pseudomonas. The diversity of pseudomonads, and the availability of genome-sequence data for multiple plant-associated Pseudomonas fluorescens, Pseudomonas mendocina, Pseudomonas putida, Pseudomonas stutzeri and Pseudomonas syringae strains, along with genome data for clinical isolates of Pseudomonas aeruginosa and for the insect pathogen Pseudomonas entomophila provide an excellent opportunity to use comparative genomic approaches to develop insight into the evolution of metabolic networks. The phylogeny connecting the seventeen genome-sequenced strains of Pseudomonas is shown in Figure 6A. The phylogeny was generated using multilocus sequencing analysis of conserved housekeeping genes (gldA, gapA, rpoD, gyrB) [23]. The metabolic network data was extracted from the KEGG database [22] on 31st January 2010 for pathway maps across the seventeen Pseudomonas strains shown in Figure 6A using the Rahnuma tool [24]. The evolution parameters were also compared between two Pseudomonas species: P. fluorescens, a saprotroph that colonizes the soil environment, and P. syringae, a plant-pathogen that is found on leaf surfaces and in plant tissues. The phylogenetic relationships between these species is shown in Figures 6B and C. The results are discussed here for the six pathway maps listed in Table 2 as they provide a representative set of different neighborhood characteristics observed across the Pseudomonas strains used in this analysis. The basic information for
each network across the seventeen Pseudomonas strains is given in Table S2.

When estimating the parameters, the hyperedges corresponding to the reactions that were common to all seventeen Pseudomonas strains were defined as core edges and the hyperedges corresponding to the reactions not present in any of these seventeen species were defined as prohibited edges. Three independent replicates of the sampler were run from random starting values for 60,000 iterations for P. fluorescens and P. syringae phylogenies, and 110,000 iterations for the phylogeny connecting the seventeen

Figure 4. Degree distributions of nodes under the neighbor-dependent and hybrid models. Boxplots showing the degree distributions of nodes obtained by simulating the evolution for the pentose phosphate pathway, lysine degradation and phenylalanine metabolism maps in P. fluorescens Pf0-1 under (A) the neighbor-dependent model and (B) the hybrid model. The red line plots the actual degree distributions observed in the corresponding pathway map in P. fluorescens Pf0-1.

doi:10.1371/journal.pcbi.1000868.g004
Pseudomonas strains with the first 10,000 iterations regarded as burn-in period in each case. The samples were collected every 10\textsuperscript{th} iteration to calculate the posterior expectations and variances of the parameters. These are listed in Table 2 and the ESS used for parameter estimation are listed in Table S3. The convergence of the algorithm was tested by checking the trace of the MCMC runs initiated from different starting values. An example is shown in Figure S5, which shows the trace for the sampler run on \textit{P. fluorescens} phylogeny (Figure 6B). The running times and the acceptance percentages of the algorithm are listed in Table S4 for all three phylogenies. We also calculated the number of insertion and deletion events for each reaction as well as at each branch of the \textit{Pseudomonas} phylogeny for all six pathway maps. These are shown in Figures S6 and S7.

The high insertion to deletion ratio (Table 2) for all three phylogenies for the glycolysis/gluconeogenesis map, pentose phosphate pathway map and pyruvate metabolism map, which are defined as a part of the carbohydrate metabolism of the bacteria in KEGG [22] and for the histidine metabolism map, which is a part of amino acid metabolism, suggests that very few reactions are missing from these networks in one or more \textit{Pseudomonas} strains used in the analysis, resulting in a highly conserved network. Lysine and phenylalanine pathway maps, on the other hand, have higher deletion rates compared to the insertion rates suggesting a variable reaction distribution across the \textit{Pseudomonas} phylogeny and instability of these functionalities. The results obtained in this study are consistent with the previous observation that the histidine metabolism map shows conservation of reactions across pseudomonads (Mithani, Hein and Preston, submitted) and that many \textit{Pseudomonas} strains are able to use histidine as sole carbon and nitrogen source [25] whereas lysine and phenylalanine pathway maps have few conserved reactions across pseudomonads (Mithani, Hein and Preston, submitted) and are poor nutrient sources for these bacteria [25]. The results also indicate that the pathway maps which are highly conserved across the seventeen \textit{Pseudomonas} strains, i.e. glycolysis/gluconeogenesis map, pentose phosphate pathway map, pyruvate metabolism map and histidine metabolism map, also have higher neighbor dependence probabilities compared to the other two pathway maps, which have variable reaction distribution across the \textit{Pseudomonas} phylogeny. This might suggest a relationship between the neighborhood structure and the conservation of networks.

The comparison of the evolution parameters between \textit{P. fluorescens} and \textit{P. syringae} provides interesting insights into the evolution of the metabolic networks of these bacteria. For example, the insertion and deletion rates are generally higher in \textit{P. fluorescens} than those in \textit{P. syringae} suggesting a higher number of insertion and deletion events in \textit{P. fluorescens} networks compared to \textit{P. syringae} networks. This was expected since the evolutionary distance between the \textit{P. fluorescens} strains is greater as compared to \textit{P. syringae} strains (Figure 6) allowing more time for the networks in \textit{P. fluorescens} to evolve. A higher deletion rate for lysine and phenylalanine pathway maps in \textit{P. syringae} compared to \textit{P. fluorescens}, however, suggests that \textit{P. syringae} have had a higher number of deletion events than \textit{P. fluorescens} during the course of evolution. This supports the finding that \textit{P. syringae} have gone through a high number of deletion events compared to the comparison between observed and expected distribution of reactions across the \textit{Pseudomonas} phylogeny, and the identification of reactions that are uniquely present or absent from a single lineage (Mithani, Hein and Preston, submitted). In addition, a very low insertion to deletion ratio (\(\lambda/\mu = 0.2564\)) for lysine metabolism in \textit{P. syringae} suggests a high number of deletion events in the lineage and consequently the loss of the ability of these bacteria to assimilate lysine. This is in agreement with nutrient utilization assays, which have reported that bacteria belonging to the species \textit{P. syringae} do not assimilate lysine as a nutrient source [25]. Phenylalanine metabolism also has a higher deletion rate as compared to insertion rate in both \textit{P. fluorescens} and \textit{P. syringae} lineages. This in conjunction with experimental data reporting the weak ability of these bacteria to utilize phenylalanine as a nutrient source might lead to a hypothesis that both \textit{P. fluorescens} and \textit{P. syringae} are drifting towards losing their ability to assimilate phenylalanine. Overall, the results show that genome reduction is taking place in plant pathogenic bacteria belonging to the species \textit{P. syringae} at a higher rate than their non-pathogenic counterparts in the species \textit{P. fluorescens}.

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure5}
\caption{Insertion Rate (\(\lambda\)) and Deletion Rate (\(\mu\)).}
\end{figure}

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure6}
\caption{Phylogenies connecting bacteria belonging to the genus \textit{Pseudomonas}.}
\end{figure}
Ancestral network reconstruction

The final aim of this study was to infer reactions present in the common ancestor of *Pseudomonas* spp. and of individual species of *Pseudomonas*. One way to address this is to predict that the common ancestor contained all the reactions that are common to existing *Pseudomonas*. The variable reactions can then be assigned using a parsimonious approach which generates a conservative model of network evolution in which a minimum number of events occur. However, the results above suggest that some lineages, particularly *P. syringae*, have undergone deletion events relative to the common ancestor and that some reactions absent in one or more modern pseudomonads might be present in the ancestral strain. To take this into account, stochastic approaches such as the Gibbs sampler described in Section ‘Estimation of parameters’ can be used to sample ancestral networks from the posterior distribution of networks and the likelihood of reactions being present at various levels of the phylogeny can be calculated.

To demonstrate this, the Gibbs sampler was run on the pathway maps listed in Table 2. The Gibbs sampler was run with the same settings that were used for parameter estimation and samples for the networks at internal nodes of the *Pseudomonas* phylogeny (Figure 6A) were collected. The degree distributions of nodes at the ancestral levels of the phylogeny are given in Figures S8, S9, S10, S11, S12, S13. The likelihood of reactions being present at each level was obtained by calculating the proportion of times each hyperedge was present in the sampled networks. The results are shown in Figures 7A–12A. Only alterable reactions, that is the reactions which were neither defined as core nor were defined as prohibited in the networks, are shown.

The ancestral network reconstruction using the Gibbs sampler reported high likelihood values for reactions which are present in all the networks down a lineage and low likelihood values for reactions which show variable distributions across the *Pseudomonas* phylogeny. For example, in the pentose phosphate pathway map (Figure 8A), the reaction R01066, which is present only in the three *P. syringae* strains, was assigned a very high likelihood of being present in the common ancestor of *P. syringae* pv. phaseolicola 1448A and *P. syringae* pv. syringae B728a as well as in the common ancestor for all the tree *P. syringae* strains but a very low likelihood of being present for all other internal networks. In contrast, R06036, which is present in sixteen out of the seventeen *Pseudomonas* strains (absent in *P. fluorescens* Pf-5), is reported to have high likelihood values of being present in all internal networks of the phylogeny.

Ancestral predictions were also generated under the parsimony model for these networks using the Fitch Algorithm [26]. When assigning the reactions at the ancestral nodes the ties were resolved in favor of presence of reactions. The results are shown in Figures 7B–12B. Reactions for which parsimony failed to resolve ancestral predictions at the root are marked with asterisks (*). Predictions generated for the *Pseudomonas* common ancestor using parsimony analysis are nearly identical to predictions generated for the *P. aeruginosa* common ancestor, which would be expected as parsimony assumes minimum evolution. In addition, parsimony generated a conservative model of network evolution in which a minimum number of events occur, but the stochastic approach takes network information into account when predicting ancestral networks.

<table>
<thead>
<tr>
<th>Pathway Map</th>
<th>Phylogeny</th>
<th>E(δ)</th>
<th>var(δ)</th>
<th>E(λ)</th>
<th>var(λ)</th>
<th>E(μ)</th>
<th>var(μ)</th>
<th>̇λ/μ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Glycolysis/Gluconeogenesis</td>
<td>(MAP00010)</td>
<td>0.2404</td>
<td>0.0070</td>
<td>1.8645</td>
<td>3.3775</td>
<td>0.7280</td>
<td>0.2891</td>
<td>2.5611</td>
</tr>
<tr>
<td>17 pseudomonads</td>
<td>0.1506</td>
<td>0.0034</td>
<td>0.7610</td>
<td>0.0117</td>
<td>0.7329</td>
<td>0.0114</td>
<td>1.0383</td>
<td></td>
</tr>
<tr>
<td>Pentose phosphate pathway</td>
<td>(MAP00030)</td>
<td>0.3251</td>
<td>0.0071</td>
<td>1.8490</td>
<td>1.6921</td>
<td>1.0172</td>
<td>0.3212</td>
<td>1.8178</td>
</tr>
<tr>
<td>17 pseudomonads</td>
<td>0.1863</td>
<td>0.0042</td>
<td>0.6762</td>
<td>0.0029</td>
<td>0.7462</td>
<td>0.0126</td>
<td>0.9062</td>
<td></td>
</tr>
<tr>
<td>Lysine degradation</td>
<td>(MAP000310)</td>
<td>0.0637</td>
<td>0.0025</td>
<td>0.6986</td>
<td>0.0103</td>
<td>2.7245</td>
<td>2.8663</td>
<td>0.2564</td>
</tr>
<tr>
<td>17 pseudomonads</td>
<td>0.0473</td>
<td>0.0030</td>
<td>0.4706</td>
<td>0.3492</td>
<td>0.6443</td>
<td>0.7188</td>
<td>0.7304</td>
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</tr>
<tr>
<td>Histidine metabolism</td>
<td>(MAP00340)</td>
<td>0.1749</td>
<td>0.0064</td>
<td>1.5321</td>
<td>0.9479</td>
<td>1.0735</td>
<td>0.3082</td>
<td>1.4272</td>
</tr>
<tr>
<td>17 pseudomonads</td>
<td>0.0986</td>
<td>0.0022</td>
<td>0.8685</td>
<td>0.0203</td>
<td>0.6795</td>
<td>0.0057</td>
<td>1.2781</td>
<td></td>
</tr>
<tr>
<td>Phenylalanine metabolism</td>
<td>(MAP00360)</td>
<td>0.0783</td>
<td>0.0029</td>
<td>1.1686</td>
<td>0.2072</td>
<td>1.8255</td>
<td>0.9345</td>
<td>0.6402</td>
</tr>
<tr>
<td>17 pseudomonads</td>
<td>0.0678</td>
<td>0.0024</td>
<td>1.0573</td>
<td>0.1448</td>
<td>2.2334</td>
<td>1.1112</td>
<td>0.4734</td>
<td></td>
</tr>
<tr>
<td>Pyruvate metabolism</td>
<td>(MAP00620)</td>
<td>0.1159</td>
<td>0.0020</td>
<td>1.5542</td>
<td>0.5376</td>
<td>1.2840</td>
<td>0.2888</td>
<td>1.2105</td>
</tr>
<tr>
<td>17 pseudomonads</td>
<td>0.1119</td>
<td>0.0007</td>
<td>0.7668</td>
<td>0.0099</td>
<td>0.6838</td>
<td>0.0142</td>
<td>1.1213</td>
<td></td>
</tr>
</tbody>
</table>

Posterior expectation and variance of parameter values (δ: neighbor dependence probability, λ: insertion rate and μ: deletion rate) estimated using the Gibbs sampler under the hybrid model. Hyperedges common to all seventeen genome-sequenced strains were defined as core and hyperedges missing in all seventeen strains were defined as prohibited hyperedges. The values are averaged over three runs of 60,000 iterations for *P. fluorescens* and *P. syringae* phylogenies, and 110,000 iterations for the phylogeny connecting the seventeen *Pseudomonas* strains with the first 10,000 iterations regarded as burn-in period in each case. Samples were collected every 10th iteration. The codes MAPxxxx correspond to the respective KEGG pathway codes [22]. Strain abbreviations: pf: *P. fluorescens* Pf-5, pfo: *P. fluorescens* Pf01-1, pfs: *P. fluorescens* SBW25, psp: *P. syringae* pv. phaseolicola 1448A, and pst: *P. syringae* pv. tomato DC3000. doi:10.1371/journal.pcbi.1000868.t002
For example, in the case of the lysine degradation map (Figure 9), six out of the ten variable reactions are reported to be absent from all the ancestral networks of the *Pseudomonas* phylogeny using the parsimony approach whereas the stochastic approach taking the reaction neighborhood data into account assigns non-zero likelihood values to these reactions for being present in the ancestral pseudomonads. Similarly, all four reactions which are predicted to be absent from all ancestral pseudomonads in the histidine metabolism map under the parsimony model have non-zero likelihoods of being present in the ancestral networks using the stochastic approach (Figure 10). The results for ancestral network reconstruction for phenylalanine metabolism (Figure 11), on the other hand, suggested a very low level of conservation of reactions across the *Pseudomonas* phylogeny using both approaches and most of the variable reactions were predicted to be absent from the common ancestor. The variable distribution of these reactions across the seventeen *Pseudomonas* strains along with the results of ancestral network reconstruction suggests that these reactions might have been gained independently at organism level.

**Figure 7. Ancestral network reconstruction for the glycolysis/gluconeogenesis map.** The ancestral networks were reconstructed over the *Pseudomonas* phylogeny shown in Figure 6A. Also shown in the bottom panel is the distribution of reactions across different *Pseudomonas* strains. (A) Likelihood of being present for alterable reactions at various levels of *Pseudomonas* phylogeny obtained by calculating the proportion of times each hyperedge was present in the networks sampled by the Gibbs sampler. (B) Reaction status obtained under maximum parsimony calculated using the Fitch algorithm [26]. When assigning the reactions at the ancestral nodes the ties were resolved in favor of presence of reactions. Reactions for which parsimony failed to resolve ancestral predictions at the root are marked with an asterisk (*). Strain abbreviations: pae: *P. aeruginosa* PAO1, pap: *P. aeruginosa* PA7, pau: *P. aeruginosa* PA14, pag: *P. aeruginosa* LESB58, pen: *P. entomophila* L48, pfi: *P. fluorescens* Pf-S, pfo: *P. fluorescens* Pf0-1, pfs: *P. fluorescens* SBW25, pmy: *P. mendocina* ymp, ppg: *P. putida* F1, ppu: *P. putida* GB-1 ppu, pp: *P. putida* KT2440, ppw: *P. putida* W619, psa: *P. stutzeri* A1501, psb: *P. syringae* pv. syringae B728a, psp: *P. syringae* pv. phaseolicola 1448A, and pst: *P. syringae* pv. tomato DC3000.

doi:10.1371/journal.pcbi.1000868.g007
In this study, we have used a Bayesian approach to study the evolution of metabolic networks. We extended the neighbor-dependent model described by Mithani et al. [9] by introducing a parameter that estimates the probability of being present in the neighbor-dependent model. This not only provides a better fit for the data but also has an advantage over the existing model since it allows one to estimate the strength of neighborhood structure affecting the evolution of given networks. It must, however, be kept in mind that inferring the neighborhood effect solely on the basis of the neighbor-dependence probability might bias the results due to the fact that a high proportion of reactions involved in central metabolism of an organism will always be present due to their functional importance. Using ortholog and synteny data in conjunction with neighbor-dependence probability would lead to better inference of the role of network structure on metabolic evolution. The idea being that if a reaction is present in most of the species that are evolutionarily close to the one being considered then it has a higher chance of being added, and if it is genetically

**Discussion**

In this study, we have used a Bayesian approach to study the evolution of metabolic networks. We extended the neighbor-dependent model described by Mithani et al. [9] by introducing a parameter that estimates the probability of being present in the neighbor-dependent model. This not only provides a better fit for the data but also has an advantage over the existing model since it allows one to estimate the strength of neighborhood structure affecting the evolution of given networks. It must, however, be kept in mind that inferring the neighborhood effect solely on the basis of the neighbor-dependence probability might bias the results due to the fact that a high proportion of reactions involved in central metabolism of an organism will always be present due to their functional importance. Using ortholog and synteny data in conjunction with neighbor-dependence probability would lead to better inference of the role of network structure on metabolic evolution. The idea being that if a reaction is present in most of the species that are evolutionarily close to the one being considered then it has a higher chance of being added, and if it is genetically

**Figure 8. Ancestral network reconstruction for the pentose phosphate pathway map.** The ancestral networks were reconstructed over the Pseudomonas phylogeny shown in Figure 6A. Also shown in the bottom panel is the distribution of reactions across different Pseudomonas strains. (A) Likelihood of being present for alterable reactions at various levels of Pseudomonas phylogeny obtained by calculating the proportion of times each hyperedge was present in the networks sampled by the Gibbs sampler. (B) Reaction status obtained under maximum parsimony calculated using the Fitch algorithm [26]. When assigning the reactions at the ancestral nodes the ties were resolved in favor of presence of reactions. Reactions for which parsimony failed to resolve ancestral predictions at the root are marked with an asterisk (*). Strain abbreviations: pae: P. aeruginosa PAO1, pap: P. aeruginosa PA7, pau: P. aeruginosa PA14, pag: P. aeruginosa LESB58, pen: P. entomophila L48, pfl: P. fluorescens Pf-5, pff: P. fluorescens P0-1, pfs: P. fluorescens SBW25, pmy: P. mendocina ymp, ppf: P. putida F1, ppb: P. putida G8-1, ppn: P. putida KT2440, ppw: P. putida W619, psa: P. stutzeri A1501, psb: P. syringae pv. phaseolicola 1448A, and pst: P. syringae pv. syringae B728a, psp: P. syringae pv. tomato DC3000. doi:10.1371/journal.pcbi.1000868.g008
linked to other reactions then they have a greater chance of being consecutively added or deleted.

The neighbor-dependent model [9] and the hybrid model described here define reaction neighborhood as reactions sharing at least one metabolite. Alternate definitions of reaction neighborhood are also possible. For example, one possible alternate is to consider the reaction directions when calculating the neighborhood and to regard two reactions as neighbors only if the metabolite connecting the two reactions is a substrate of one and the product of the other. Similarly, it is also possible to use other measures such as sequence similarity [27–29] or network distance measures [30,31] in conjunction with the network structure to model the evolution of metabolic networks. There are, however, limitations associated with the models of metabolic evolution solely based on network structure and sequence similarity. There are a number of other factors affecting metabolic evolution. These include substrate availability (for example, availability of a new nutrient in the environment may favor the insertion of reactions which bring this new metabolite into the mainstream metabolism), gene expression (for example, a decrease in the gene expression

Figure 9. Ancestral network reconstruction for the lysine degradation map. The ancestral networks were reconstructed over the Pseudomonas phylogeny shown in Figure 6A. Also shown in the bottom panel is the distribution of reactions across different Pseudomonas strains. (A) Likelihood of being present for alterable reactions at various levels of Pseudomonas phylogeny obtained by calculating the proportion of times each hyperedge was present in the networks sampled by the Gibbs sampler. (B) Reaction status obtained under maximum parsimony calculated using the Fitch algorithm [26]. When assigning the reactions at the ancestral nodes the ties were resolved in favor of presence of reactions. Strain abbreviations: pae: P. aeruginosa PA01, pap: P. aeruginosa PA7, pau: P. aeruginosa PA14, pag: P. aeruginosa LESB58, pen: P. entomophila L48, pfo: P. fluorescens Pf-5, pfl: P. fluorescens SBW25, pps: P. mendocina ymp, pff: P. putida F1, ppg: P. putida GB-1 pps: P. putida KT2440, ppw: P. putida W619, psa: P. stutzeri A1501, psp: P. syringae pv. syringae B728a, psp: P. syringae pv. phaseolicola 1448A, and pst: P. syringae pv. tomato DC3000. doi:10.1371/journal.pcbi.1000868.g009
relating to an enzyme catalyzing a reaction may force the metabolic network to find alternate routes) and reaction mechanism (for example, a reaction which is chemically inefficient may be favored for deletion compared to an efficient reaction). Factors such as these must be taken into account when modeling the evolution of metabolic networks to depict a more realistic picture of evolution.

We also presented a Gibbs sampler to sample the networks at internal nodes of a phylogenetic tree where the internal networks were sampled by conditioning on three neighbors (one parent and two children) in an approach similar to the one used by Holmes and Bruno [18] for DNA sequence alignment. The sampler considered the network structure surrounding the hyperedge being sampled in addition to the state of the hyperedge in the three neighboring networks when calculating the new state thus resulting in an informed sampling procedure. When sampling ancestral networks, it was assumed that all sampled networks were valid networks. However, not all networks may be functionally viable. For example, a network might not be able to produce a key metabolite which is required or may result in disconnected networks.

Figure 10. Ancestral network reconstruction for the histidine metabolism map. The ancestral networks were reconstructed over the Pseudomonas phylogeny shown in Figure 6A. Also shown in the bottom panel is the distribution of reactions across different Pseudomonas strains. (A) Likelihood of being present for alterable reactions at various levels of Pseudomonas phylogeny obtained by calculating the proportion of times each hyperedge was present in the networks sampled by the Gibbs sampler. (B) Reaction status obtained under maximum parsimony calculated using the Fitch algorithm [26]. When assigning the reactions at the ancestral nodes the ties were resolved in favor of presence of reactions. Strain abbreviations: pae: P. aeruginosa PAO1, pap: P. aeruginosa PA7, pau: P. aeruginosa PA14, pag: P. aeruginosa LESB58, pen: P. entomophila L48, pfs: P. fluorescens PFO-1, pff: P. fluorescens SBW25, pmy: P. mendocina ymp, ppp: P. putida F1, pp: P. putida GB-1 ppu: P. putida KT2440, ppw: P. putida W619, psa: P. stutzeri A1501, psb: P. syringae pv. syringae B728a, psp: P. syringae pv. phaseolicola 1448A, and pst: P. syringae pv. tomato DC3000. doi:10.1371/journal.pcbi.1000868.g010
components that compromise network functionality. Checking for validity of networks occurring at ancestral nodes is an important area for further research.

A Gibbs sampler to estimate the evolution parameters was also presented. Standard distributions were used to generate proposals for parameters. The standard distributions provide satisfactory mixing of the MCMC sampler with appropriate scaling [32]. The rate parameters were sampled from a gamma distribution where scale and shape parameters were calculated from the current network and the proposals for neighbor dependence probability were generated using a beta distribution with its scale parameters calculated from the networks present at the leaves of the given phylogenetic tree. A uniform prior was used when estimating the parameters, which assigns equal probability to each point in the parameter space. It might be useful to explore the dependence between the number of insertions and deletions on the given phylogeny and to investigate the use of other prior distributions. Besides this, when calculating the likelihood of evolution, it was assumed that the phylogenetic tree was known through sequence analysis. This simplified the problem by not requiring a sum over the

**Figure 11. Ancestral network reconstruction for the phenylalanine metabolism map.** The ancestral networks were reconstructed over the *Pseudomonas* phylogeny shown in Figure 6A. Also shown in the bottom panel is the distribution of reactions across different *Pseudomonas* strains. (A) Likelihood of being present for alterable reactions at various levels of *Pseudomonas* phylogeny obtained by calculating the proportion of times each hyperedge was present in the networks sampled by the Gibbs sampler. (B) Reaction status obtained under maximum parsimony calculated using the Fitch algorithm [26]. When assigning the reactions at the ancestral nodes the ties were resolved in favor of presence of reactions. Strain abbreviations: pae: *P. aeruginosa* PAO1, pap: *P. aeruginosa* PA7, pau: *P. aeruginosa* PA14, pag: *P. aeruginosa* LESB58, pen: *P. entomophila* L48, pfl: *P. fluorescens* SBW25, pmy: *P. mendocina* ymp, ppt: *P. putida* GB-1 ppb, P. putida KT2440, pps: *P. putida* W619, psa: *P. stutzeri* A1501, psb: *P. syringae* pv. syringae B728a, psp: *P. syringae* pv. phaseolicola 1448A, and pst: *P. syringae* pv. tomato DC3000. doi:10.1371/journal.pcbi.1000868.g011
all possible branch lengths. However, when calculating the tree using sequence data, the branch lengths depend on the set of genes used for generating the tree and their evolutionary distances. Thus, different set of genes used could result in different branch lengths. To be able to make useful inferences using an evolutionary model such as the one described here, this uncertainty in the tree must be taken into account by summing over all possible branch lengths. In addition, the effects of using a phylogenetic tree constructed de novo from metabolic networks [30,31] on the model need to be further explored.

The evolution parameters were estimated on a phylogeny connecting the metabolic networks of bacteria belonging to the genus *Pseudomonas* using the Gibbs sampler. The likelihood values for reactions to be present at various levels of *Pseudomonas* phylogeny were calculated using the networks sampled by the Gibbs sampler and compared to those obtained using parsimony. The stochastic assignment of reactions in ancestral networks offers an edge over deterministic approaches like parsimony which provides the minimum number of transformations required to explain the evolution of a reaction.
of the tree and can, therefore, underestimate the total number of changes. In addition, using the MCMC approach based on neighborhood dependence takes network structure into account, and may be particularly useful in resolving ancestral predictions at the root of phylogenies, or in situations where parsimony is unable to assign states unambiguously [see Figures 7 and 8].

An important factor affecting the results when estimating the evolution parameters and reconstructing the ancestral networks relates to the use of individual pathway maps. Although computationally tractable, individual pathway maps do not take a complete network perspective and may, therefore, lead to incorrect results by ignoring a part of the reaction neighborhood, the so-called border effect. This is particularly true for reactions which occur at the boundary of a metabolic pathway map, which may have a large number of their neighbors not included in that pathway map. The calculation of reaction neighborhood solely using the pathway map under consideration ignores all neighboring reactions that are not present in the pathway map thus affecting the likelihood values. For example, consider R01424 in phenylalanine metabolism. This reaction is present in thirteen out of seventeen pseudomonads including all three P. syringae strains, all four P. putida strains and two out of the three P. fluorescens strains. It was, therefore, expected that the reaction would have a high likelihood of being present in the common ancestor of P. fluorescens, P. syringae and P. putida but, on contrary, was reported to have a relatively low likelihood at this level (Figure 11). Closer inspection of the reaction revealed that it links the phenylalanine metabolism to the pathway map relating to benzoate degradation via coenzyme A and has neighbors spanning across multiple pathway maps. Phenylalanine pathway map contains only 2 neighboring reactions of the reaction R01424 whereas using the data from all pathway maps relating to metabolism results in 53 neighbors. Thus, evaluating the likelihood of R01424 at ancestral levels of the Pseudomonas phylogeny solely on the basis of reactions involved in phenylalanine metabolism leads to a very poor neighborhood surrounding the reaction and, consequently, weights down the presence of the reaction in the common ancestor resulting in a low likelihood value. A possible solution to overcome this border effect is to use the full network structure when calculating reaction neighborhoods. However, the computational feasibility of using full network structure when calculating reaction neighborhoods requires further investigation.

When performing the analyses the hyperedges present in all seventeen Pseudomonas strains were defined as core and the hyperedges missing from all the strains were defined as prohibited hyperedges. However, the results presented in this analysis suggest that pathogenic bacteria belonging to species P. syringae have gone through a high number of deletion events compared to other species. Assigning core edges solely on the basis of intersection model may, therefore, bias the results towards the loss of reactions which might be essential in non-pathogenic bacteria. Similarly, prohibiting reactions that are not present in any one of the seventeen genome-sequenced strains would prevent the common ancestor from having reactions which might have been lost very early during the course of evolution. To model scenarios like these the provision of having a lineage specific core and prohibited hyperedges must be explored. Alternatively, it might be useful to assign core and prohibited hyperedges using the ortholog data from closely related bacteria, or by incorporating metabolic information from organisms sharing the same environment in the set of permitted reactions. Comparing ancestral network predictions generated using different set of core and prohibited hyperedges might provide clues about the functionality of the common ancestors of the bacteria and the environment the ancestors might have colonized.

Finally, the analysis presented here uses data from the KEGG database. The metabolic annotations available for the majority of genome-sequenced organisms are generated using automated annotation tools based on the similarity of predicted genes to genes of known function and therefore contain a substantial amount of noise. For example, some genes predicted to have a broad enzymatic function are linked to multiple reactions, while others fail to meet the detection threshold for annotation and are therefore recorded as absent. Nevertheless, networks deposited in databases like KEGG are commonly treated as if they are as certain as sequence data, which is a serious error that undermines many present investigations. It would be desirable to take this noise into account while modeling the evolution of metabolic networks. One way would be to use hidden states to model experimentally validate metabolisms which are observed though predicted metabolisms. This will not only enable one to model the noise in the data but also allow correct prediction of a metabolism for an organism using homologous information similar to comparative genome annotation [33].

In summary, evolutionary modeling of metabolic network is an important area. Using statistical models of network evolution such as the one described here not only allow one to investigate how the metabolic networks evolve in closely related organisms but also enable testing of biological hypotheses such as specialization of genomes and identification of regions of metabolic networks that are under high selection.

**Supporting Information**

**Figure S1** Simulation results for insertion frequencies for the toy network H1 shown in Figure 1 using hybrid model for different values of δ. Also shown in the top panel are the number of neighbors for each hyperedge based on the reference network.

**Figure S2** Sub-networks of the toy network H1 shown in Figure 1 for different sub-network sizes (N) obtained by iteratively dividing the network on the basis of neighborhood. The hyperedges which were originally absent from H1 but present in the sub-network are shown in gray.

**Figure S3** Likelihood surfaces calculated by matrix exponentiation using all 1024 networks (True Likelihood) and using the networks visited by the Gibbs sampler (Estimated Likelihood) for different insertion and deletion rates for the toy networks phylogeny shown in Figure 1. The true and estimated maximum likelihood values are marked with asterisks. The maximum likelihood value was estimated using the Gibbs sampler for parameter estimation.

**Figure S4** An example MCMC trace showing the rate parameters for the first 1,000 iterations of the Gibbs sampler for the toy networks phylogeny shown in Figure 1.

**Figure S5** Example MCMC traces showing the rate parameters for the first 1,000 iterations of the Gibbs sampler initiated from different starting values. The sampler was run on the Pseudomonas fluorescens phylogeny shown in Figure 6B for different metabolic networks.
Figure S6 Number of insertion and deletion events for the alterable reactions, that is the reactions which were neither defined as core nor were defined as prohibited in the network obtained using the Gibbs sampler run under the hybrid model. The sampler was run for the six pathway maps used in this analysis over the phylogeny connecting the seventeen *Pseudomonas* strains shown in Figure 6A for 110,000 iterations with the first 10,000 iterations regarded as burn-in period. Samples were collected every 10th iteration. Found at: doi:10.1371/journal.pcbi.1000868.s006 (0.02 MB PDF)

Figure S7 Number of insertion and deletion events at each branch of the phylogeny connecting the seventeen *Pseudomonas* strains shown in Figure 6A obtained using the Gibbs sampler run under the hybrid model. The sampler was run for 110,000 iterations with the first 10,000 iterations regarded as burn-in period. Samples were collected every 10th iteration. Strain abbreviations: *P. aeruginosa* PAO1, *P. aeruginosa* PA7, *P. aeruginosa* PA14, *P. aeruginosa* LESB58, *P. entomopathica* L48, *P. fluorescens* P-5, *P. fluorescens* P-0-1, *P. fluorescens* SBW25, *P. mendocina* ymp, *P. putida* F1, *P. putida* GB-1, *P. putida* KT2440, *P. putida* W619, *P. stutzeri* A1501, *P. syringae* pv. syringae B728a, *P. syringae* pv. phaseolicola 1448A, and *P. syringae* pv. tomato DC3000. Found at: doi:10.1371/journal.pcbi.1000868.s007 (0.02 MB PDF)

Figure S8 Degree distributions of nodes at the ancestral levels of the *Pseudomonas* phylogeny shown in Figure 6A for the glycolysis/glucoseonogenesis map obtained using the Gibbs sampler. The actual degree distributions observed for the seventeen genome-sequenced *Pseudomonas* strains are shown in red. Strain abbreviations: *P. aeruginosa* PAO1, *P. aeruginosa* PA7, *P. aeruginosa* PA14, *P. aeruginosa* LESB58, *P. entomopathica* L48, *P. fluorescens* P-5, *P. fluorescens* P-0-1, *P. fluorescens* SBW25, *P. mendocina* ymp, *P. putida* F1, *P. putida* GB-1, *P. putida* KT2440, *P. putida* W619, *P. stutzeri* A1501, *P. syringae* pv. syringae B728a, *P. syringae* pv. phaseolicola 1448A, and *P. syringae* pv. tomato DC3000. Found at: doi:10.1371/journal.pcbi.1000868.s008 (0.04 MB PDF)

Figure S9 Degree distributions of nodes at the ancestral levels of the *Pseudomonas* phylogeny shown in Figure 6A for the pentose phosphate pathway map obtained using the Gibbs sampler. The actual degree distributions observed for the seventeen genome-sequenced *Pseudomonas* strains are shown in red. Strain abbreviations: *P. aeruginosa* PAO1, *P. aeruginosa* PA7, *P. aeruginosa* PA14, *P. aeruginosa* LESB58, *P. entomopathica* L48, *P. fluorescens* P-5, *P. fluorescens* P-0-1, *P. fluorescens* SBW25, *P. mendocina* ymp, *P. putida* F1, *P. putida* GB-1, *P. putida* KT2440, *P. putida* W619, *P. stutzeri* A1501, *P. syringae* pv. syringae B728a, *P. syringae* pv. phaseolicola 1448A, and *P. syringae* pv. tomato DC3000. Found at: doi:10.1371/journal.pcbi.1000868.s009 (0.07 MB PDF)

Figure S10 Degree distributions of nodes at the ancestral levels of the *Pseudomonas* phylogeny shown in Figure 6A for the lysine degradation map obtained using the Gibbs sampler. The actual degree distributions observed for the seventeen genome-sequenced *Pseudomonas* strains are shown in red. Strain abbreviations: *P. aeruginosa* PAO1, *P. aeruginosa* PA7, *P. aeruginosa* PA14, *P. aeruginosa* LESB58, *P. entomopathica* L48, *P. fluorescens* P-5, *P. fluorescens* P-0-1, *P. fluorescens* SBW25, *P. mendocina* ymp, *P. putida* F1, *P. putida* GB-1, *P. putida* KT2440, *P. putida* W619, *P. stutzeri* A1501, *P. syringae* pv. syringae B728a, *P. syringae* pv. phaseolicola 1448A, and *P. syringae* pv. tomato DC3000.

Found at: doi:10.1371/journal.pcbi.1000868.s010 (0.08 MB PDF)

Figure S11 Degree distributions of nodes at the ancestral levels of the *Pseudomonas* phylogeny shown in Figure 6A for the histidine metabolism map obtained using the Gibbs sampler. The actual degree distributions observed for the seventeen genome-sequenced *Pseudomonas* strains are shown in red. Strain abbreviations: *P. aeruginosa* PAO1, *P. aeruginosa* PA7, *P. aeruginosa* PA14, *P. aeruginosa* LESB58, *P. entomopathica* L48, *P. fluorescens* P-5, *P. fluorescens* P-0-1, *P. fluorescens* SBW25, *P. mendocina* ymp, *P. putida* F1, *P. putida* GB-1, *P. putida* KT2440, *P. putida* W619, *P. stutzeri* A1501, *P. syringae* pv. syringae B728a, *P. syringae* pv. phaseolicola 1448A, and *P. syringae* pv. tomato DC3000. Found at: doi:10.1371/journal.pcbi.1000868.s011 (0.04 MB PDF)

Figure S12 Degree distributions of nodes at the ancestral levels of the *Pseudomonas* phylogeny shown in Figure 6A for the phenylalanine metabolism map obtained using the Gibebsampler. The actual degree distributions observed for the seventeen genome-sequenced *Pseudomonas* strains are shown in red. Strain abbreviations: *P. aeruginosa* PAO1, *P. aeruginosa* PA7, *P. aeruginosa* PA14, *P. aeruginosa* LESB58, *P. entomopathica* L48, *P. fluorescens* P-5, *P. fluorescens* P-0-1, *P. fluorescens* SBW25, *P. mendocina* ymp, *P. putida* F1, *P. putida* GB-1, *P. putida* KT2440, *P. putida* W619, *P. stutzeri* A1501, *P. syringae* pv. syringae B728a, *P. syringae* pv. phaseolicola 1448A, and *P. syringae* pv. tomato DC3000. Found at: doi:10.1371/journal.pcbi.1000868.s012 (0.03 MB PDF)

Figure S13 Degree distributions of nodes at the ancestral levels of the *Pseudomonas* phylogeny shown in Figure 6A for the pyruvate metabolism map obtained using the Gibbs sampler. The actual degree distributions observed for the seventeen genome-sequenced *Pseudomonas* strains are shown in red. Strain abbreviations: *P. aeruginosa* PAO1, *P. aeruginosa* PA7, *P. aeruginosa* PA14, *P. aeruginosa* LESB58, *P. entomopathica* L48, *P. fluorescens* P-5, *P. fluorescens* P-0-1, *P. fluorescens* SBW25, *P. mendocina* ymp, *P. putida* F1, *P. putida* GB-1, *P. putida* KT2440, *P. putida* W619, *P. stutzeri* A1501, *P. syringae* pv. syringae B728a, *P. syringae* pv. phaseolicola 1448A, and *P. syringae* pv. tomato DC3000. Found at: doi:10.1371/journal.pcbi.1000868.s013 (0.08 MB PDF)

Table S1 Pseudo-likelihood $P^{nH}_H[H]_H$ conditioned on $H$ for the toy networks shown in Figure 1 using different sub-network sizes. Also shown are the exact values calculated by full network exponentiation and by using the MCMC approach based on path sampling described in Mihani et al. [9] with 11,000 iterations including the first 1,000 iteration as burn-in period. Found at: doi:10.1371/journal.pcbi.1000868.s014 (0.03 MB PDF)

Table S2 Basic information of the metabolic networks for the seventeen genome-sequenced strains of *Pseudomonas* used in this study. A reversible reaction was represented by two hyperedges (one in either direction) in this study. The codes MAPxxxxx correspond to the respective KEGG pathway codes [22]. Found at: doi:10.1371/journal.pcbi.1000868.s015 (0.04 MB PDF)

Table S3 Effective sample sizes (ESS) for the estimated parameters ($\delta$: neighbor dependence probability, $\lambda$: insertion rate and $\mu$: deletion rate) using the Gibbs sampler run under the hybrid model for the evolution of metabolic networks over the phylogeny connecting different *Pseudomonas* strains (Figure 6). Hyperedges that were common to all seventeen strains were regarded as core edges and hyperedges missing in all seventeen strains were regarded as prohibited edges. The values are averaged across
three runs of 60,000 iterations for *P. fluorescens* and *P. syringae* phylogenies, and 110,000 iterations for the phylogeny connecting the seventeen *Pseudomonas* strains with the first 10,000 iterations regarded as burn-in period in each case. Samples were collected every 10th iteration. Strain abbreviations: pfl: *Pseudomonas fluorescens* Pf-5, psf: *Pseudomonas fluorescens* PFO-1, psb: *Pseudomonas syringae* pv. syringae, pv. tomato DC3000, pv. phaseolicola 1448A, psb: *Pseudomonas syringae* pv. syringae B728a, psp: *Pseudomonas syringae* pv. phaseolicola 1448A, and pst: *Pseudomonas syringae* pv. tomato DC3000.

Found at: doi:10.1371/journal.pcbi.1000868.s017 (0.03 MB PDF)

**Table S4** Running time and the acceptance percentage of the Gibbs sampler for the estimation of evolution parameters (δ: neighbor dependence probability, λ: insertion rate and μ: deletion rate) run under the hybrid model for different metabolic networks over the phylogeny connecting different *Pseudomonas* strains (Figure 6). Hyperedges that were common to all seventeen strains were regarded as core edges and hyperedges missing in all seventeen strains were regarded as prohibited edges. The values are averaged across three runs of 60,000 iterations for *P. fluorescens* and *P. syringae* phylogenies, and 110,000 iterations for the phylogeny connecting the seventeen *Pseudomonas* strains with the first 10,000 iterations regarded as burn-in period in each case.

**References**


