Dynamics of vector-borne pathogens

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Infectious diseases

Transmission

Mode of transmission

Direct transmission

Direct contact

Droplet spread

Indirect transmission

Airborne

Vehicle-borne (fomites)

Vector-borne (mechanical or biological)

Portal of entry

Portal of exit

Infectious diseases

Transmission

Mode of transmission

<u>Direct transmission</u>

Direct contact

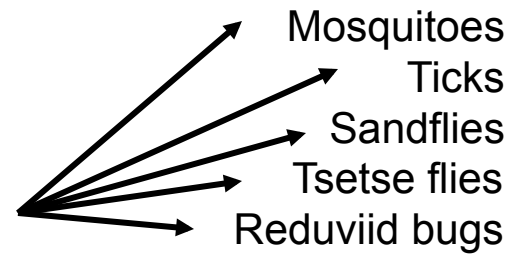
Droplet spread

<u>Indirect transmission</u>

Airborne

Vehicle-borne (fomites)

Vector-borne (mechanical or biological)

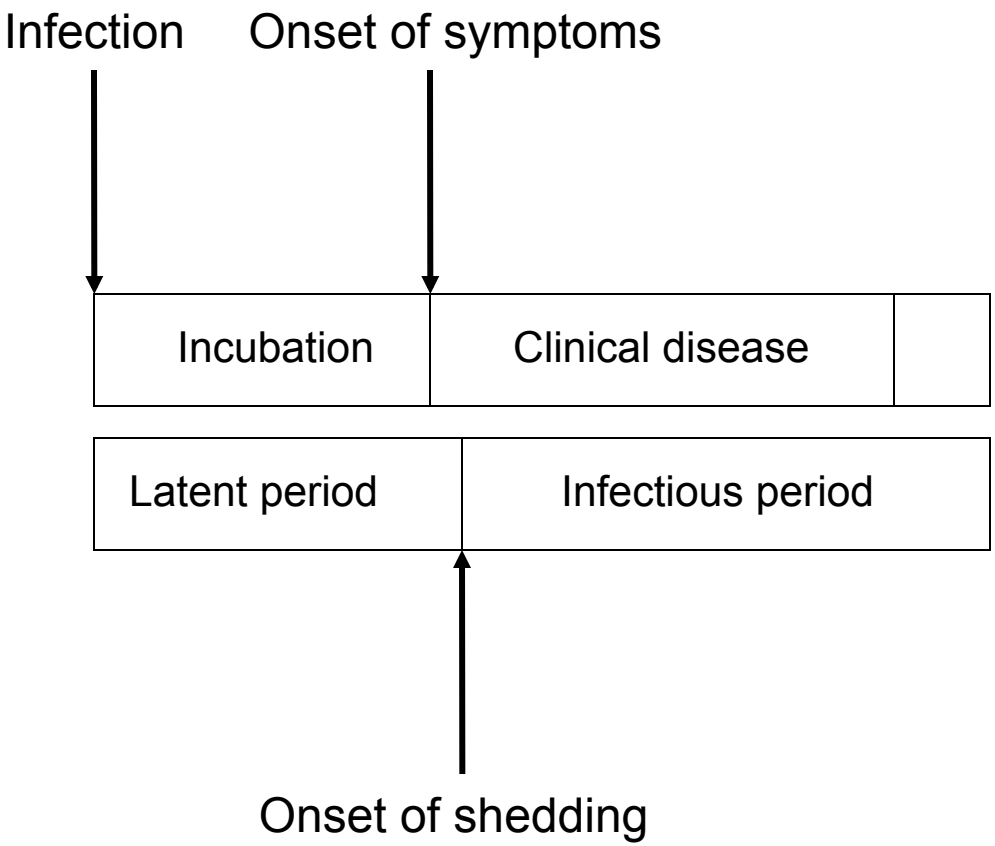


Portal of entry

Portal of exit

Vector-borne pathogens

“ ”



Vector-borne pathogens

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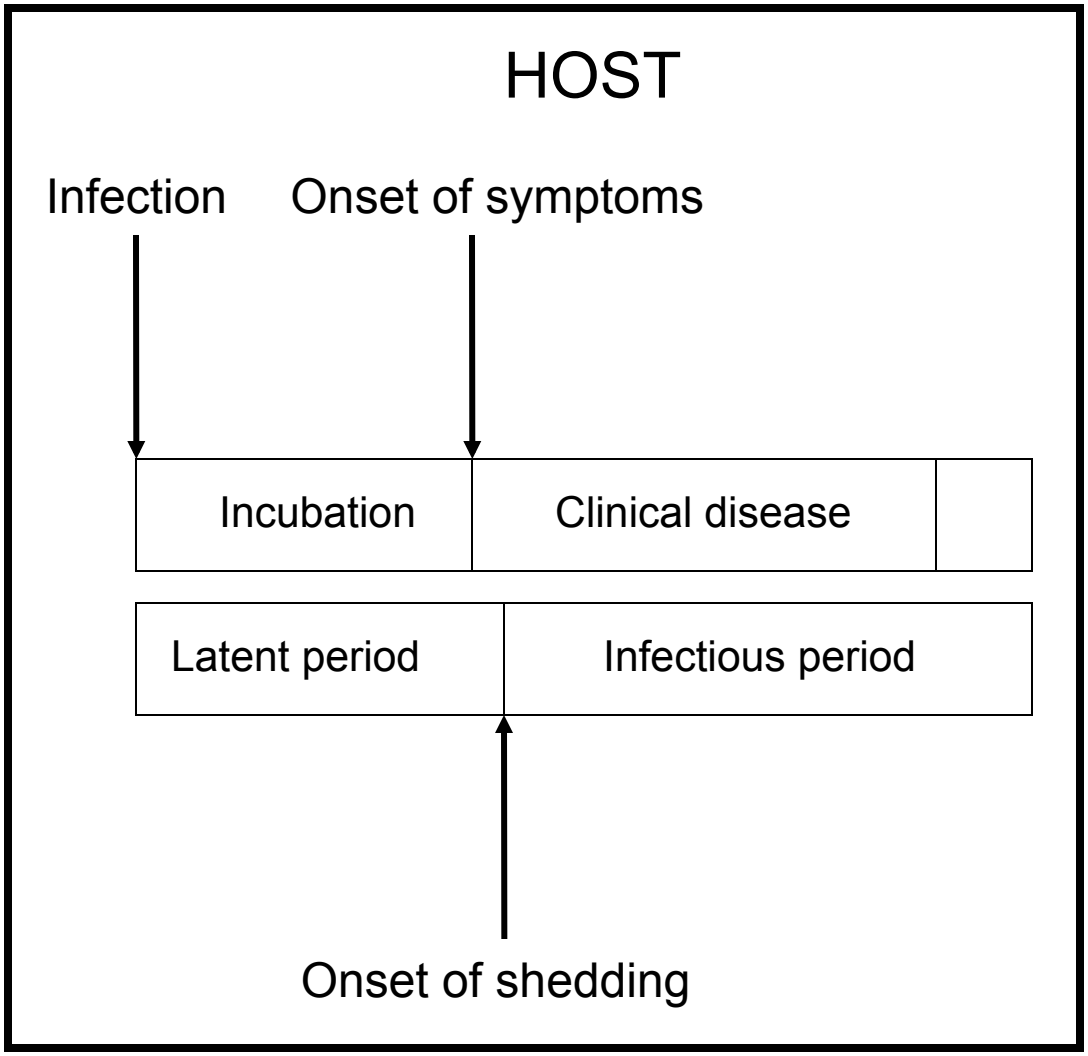
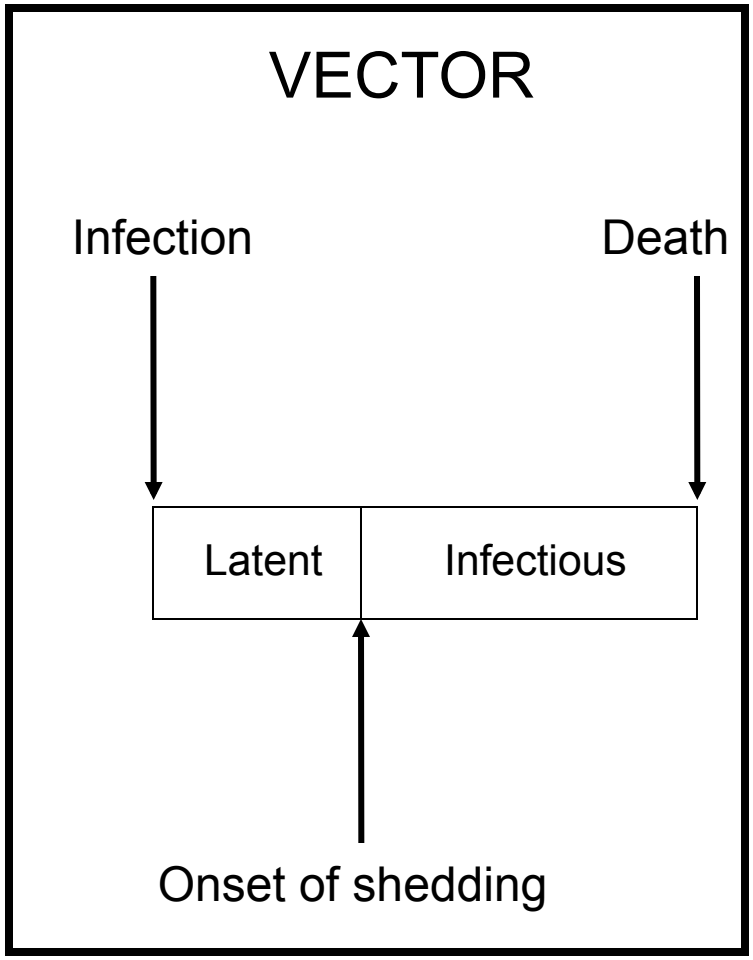




Vector-borne pathogens

“ ”

Often acute:

timecourse of infection <<
normal lifespan of host

BUT

timecourse of infection ~
normal lifespan of vector

Sometimes immunizing:

infection may stimulate
antibody production,
preventing future
infection…

or may not…

or somewhere in between

Vector-borne pathogens

Examples

Mosquitoes

Anopheles spp., malaria vectors

Culex spp., West Nile vectors

Other biting flies

*Phlebotomus papatas*i, *Leishmania* vector

Glossina spp., African trypanosomiasis vectors

True bugs

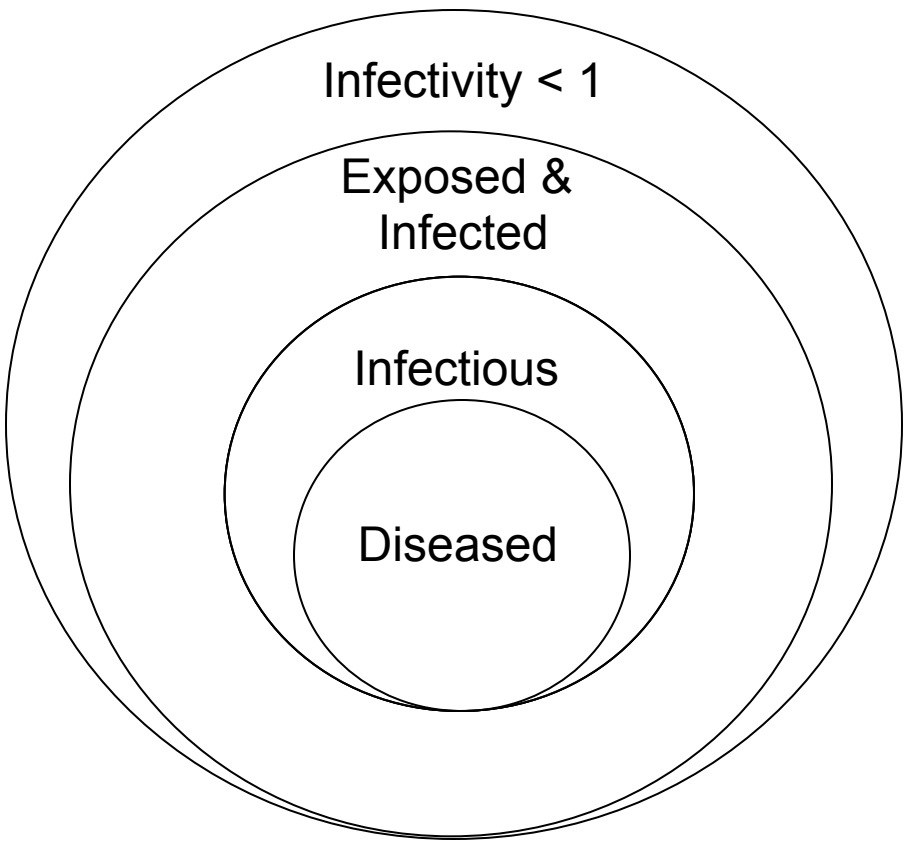
Triatoma infestans, Chagas vector

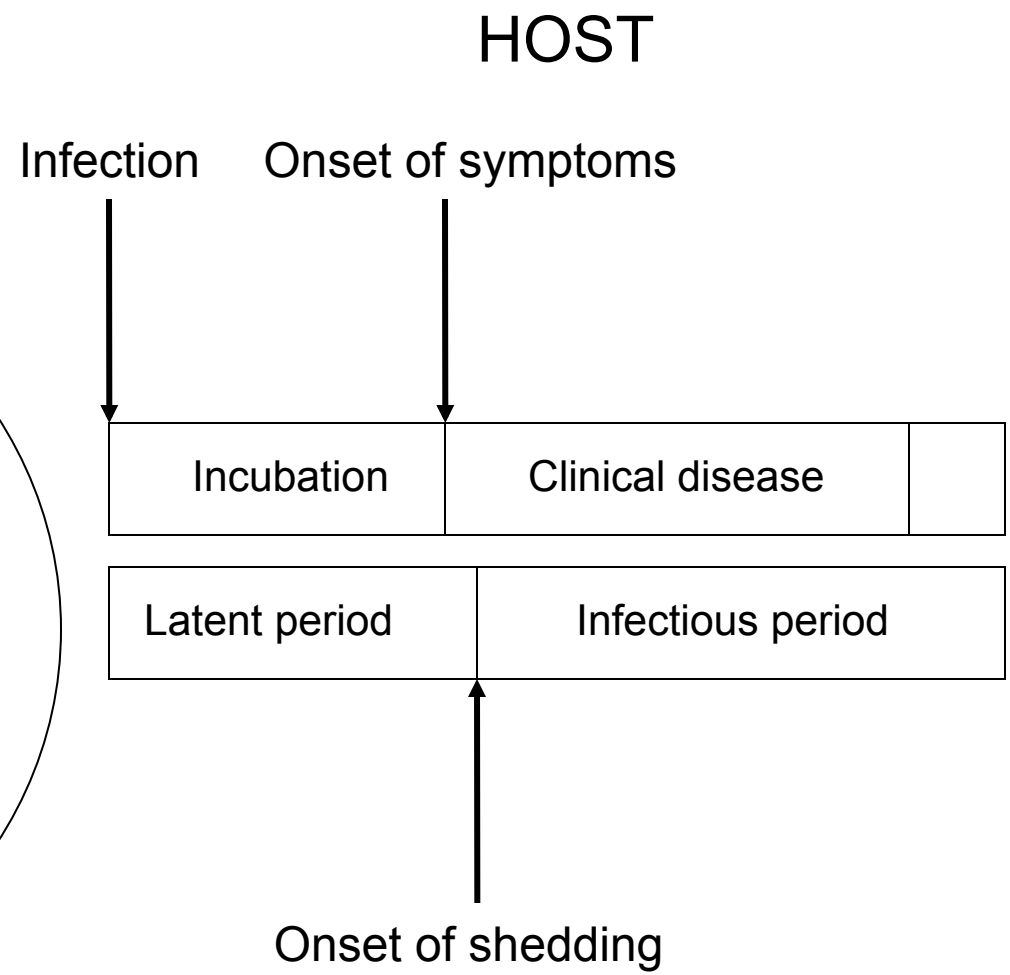
Ticks

Amblyomma spp., heartwater vectors

Vector-borne pathogens

not so A simple view of the world

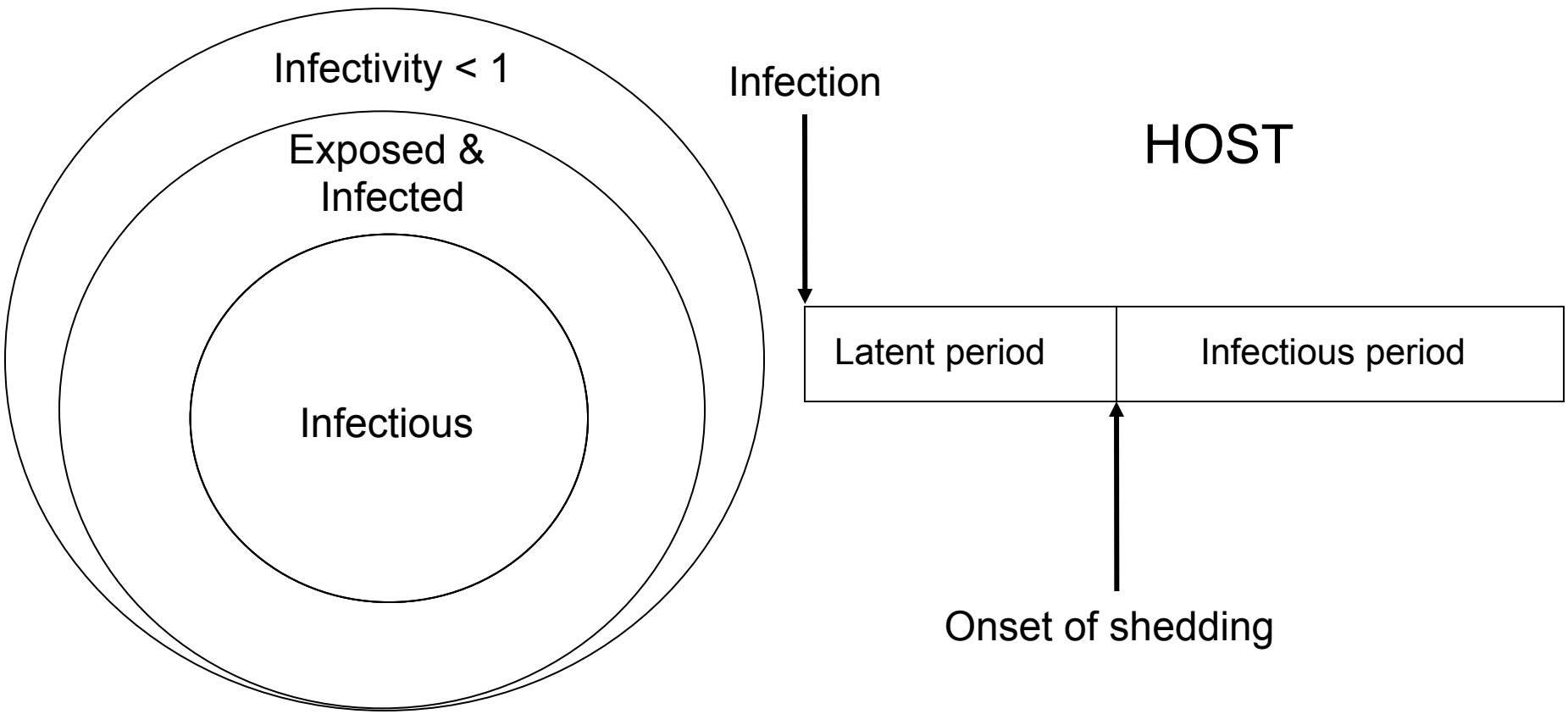




Vector-borne pathogens

not so A simple view of the world

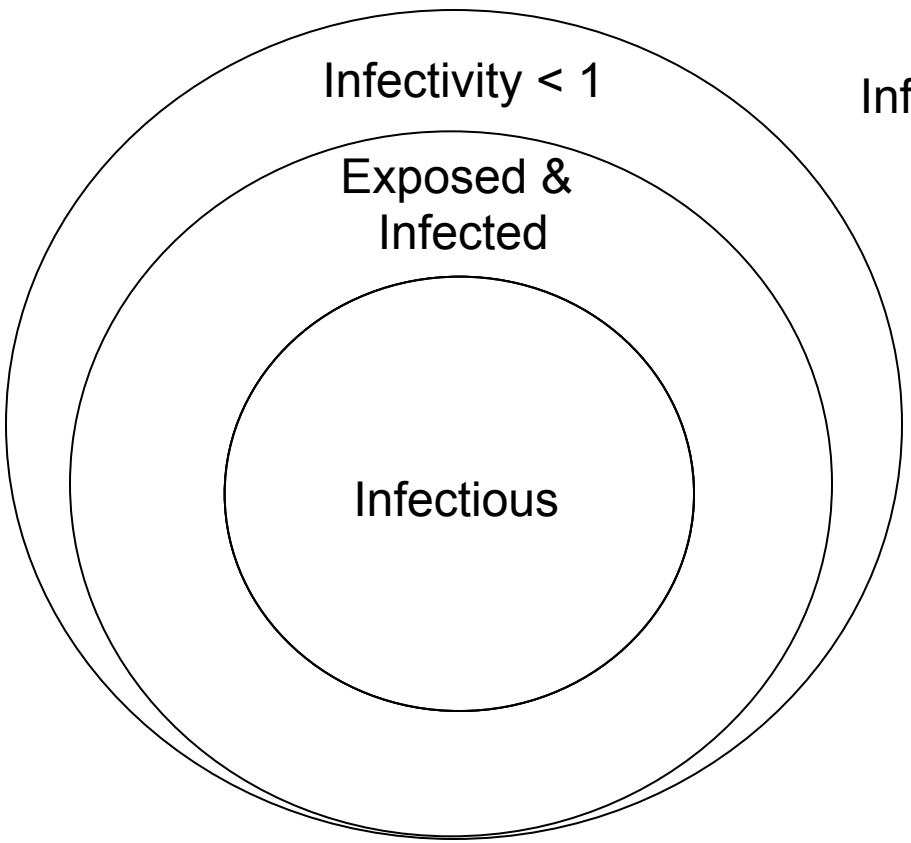
Don t worry about symptoms and disease!

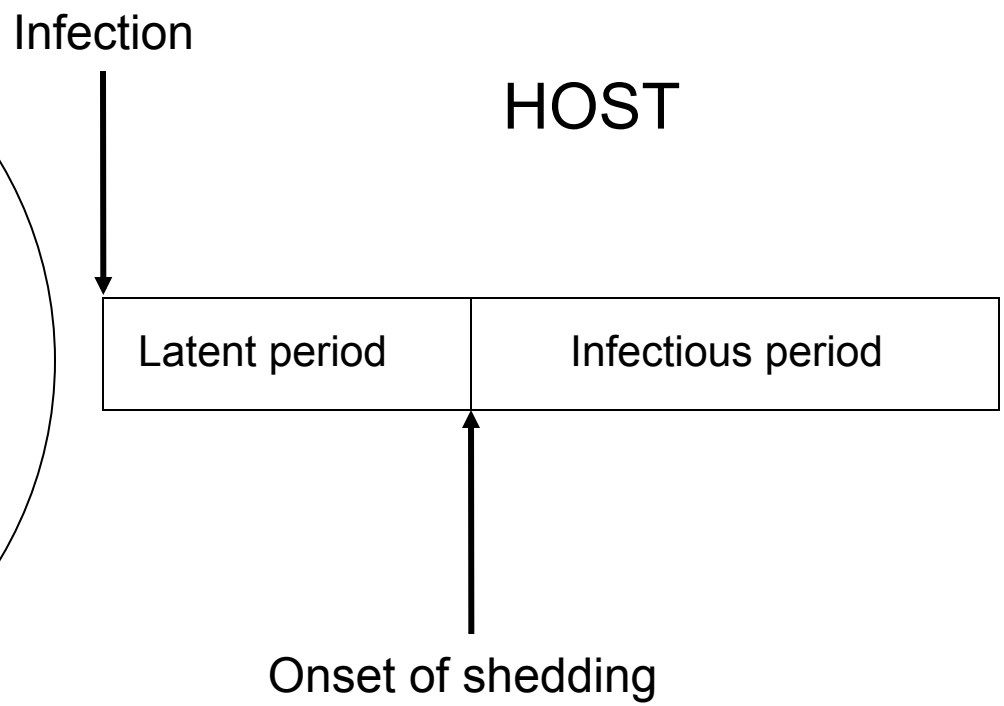


Vector-borne pathogens

not so A simple view of the world

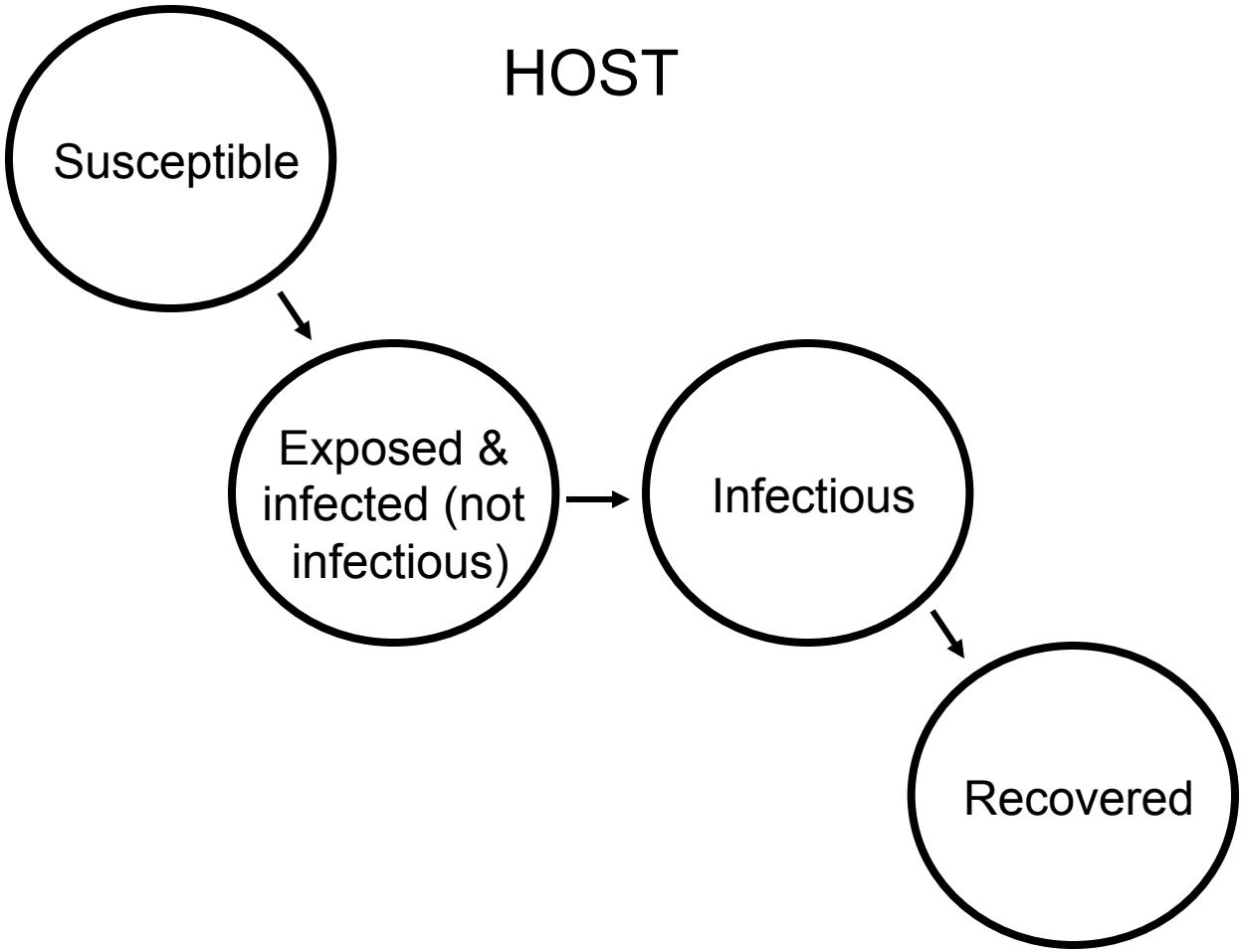
β_H = infectivity to humans x per capita (vector) biting rate





Vector-borne pathogens

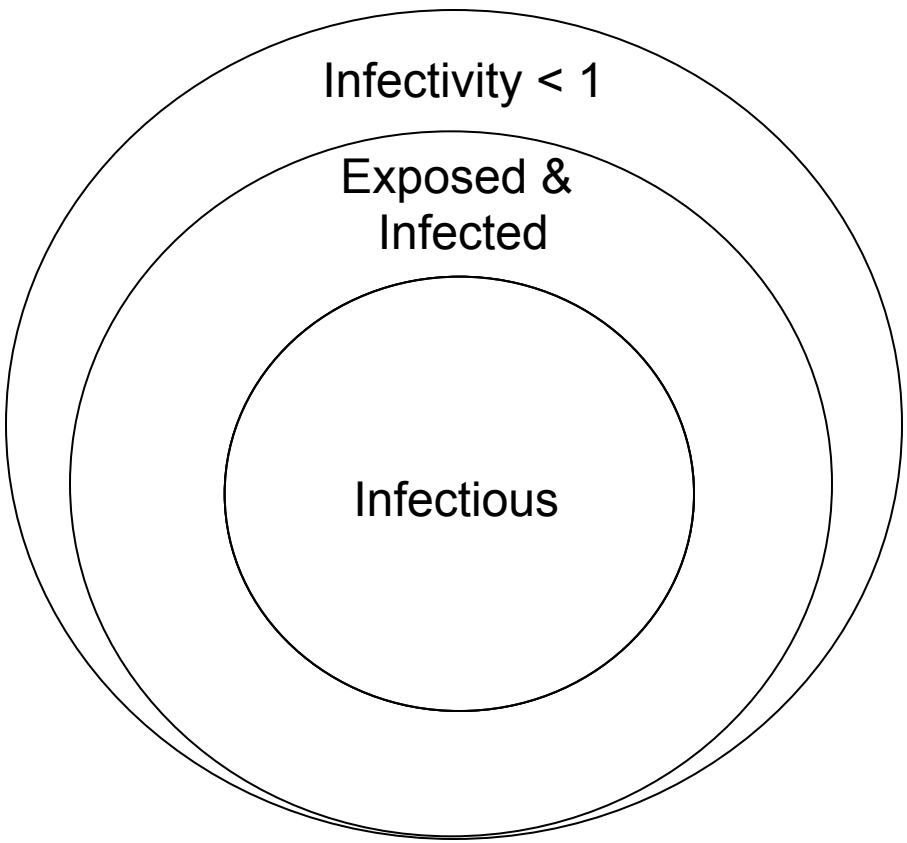
not so A simple view of the world



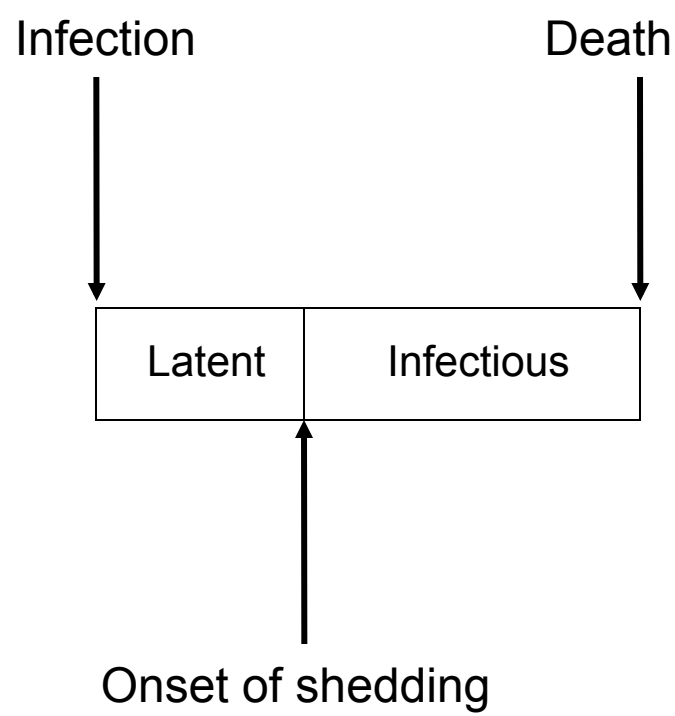
Vector-borne pathogens

not so A simple view of the world

β_V = infectivity to vectors x per capita (vector) biting rate

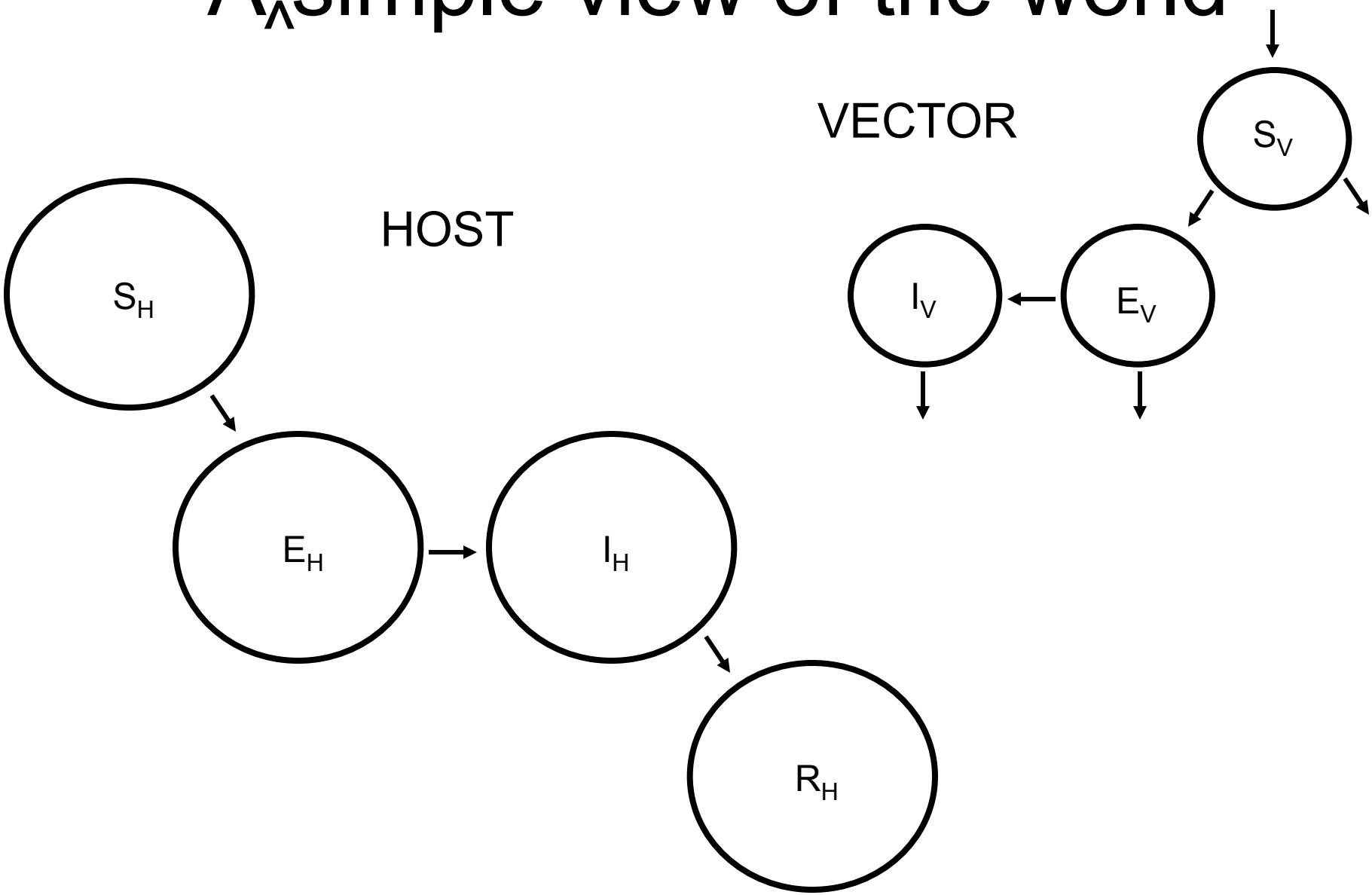


VECTOR



Vector-borne pathogens

not so A simple view of the world



Vector-borne pathogens

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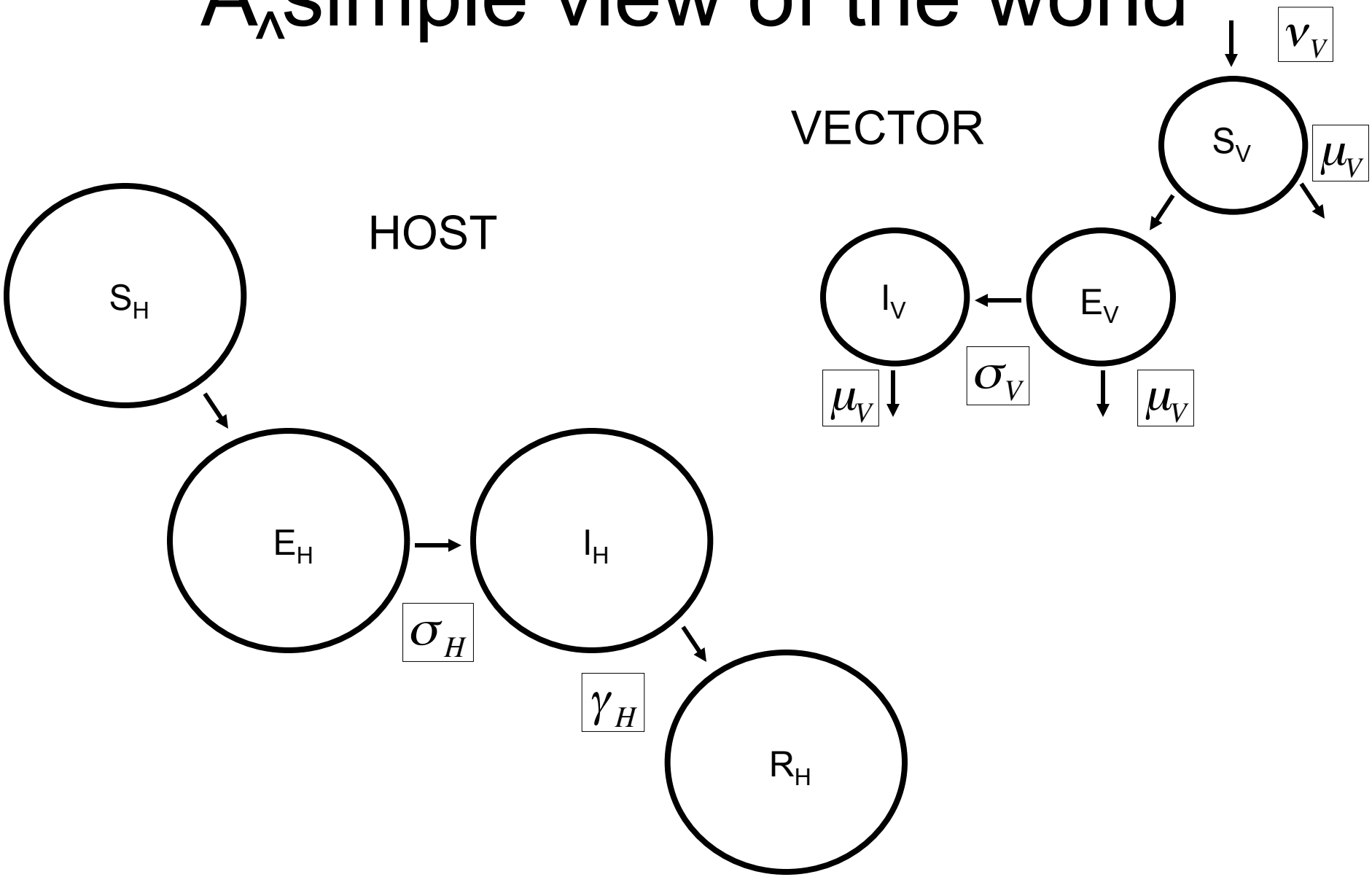
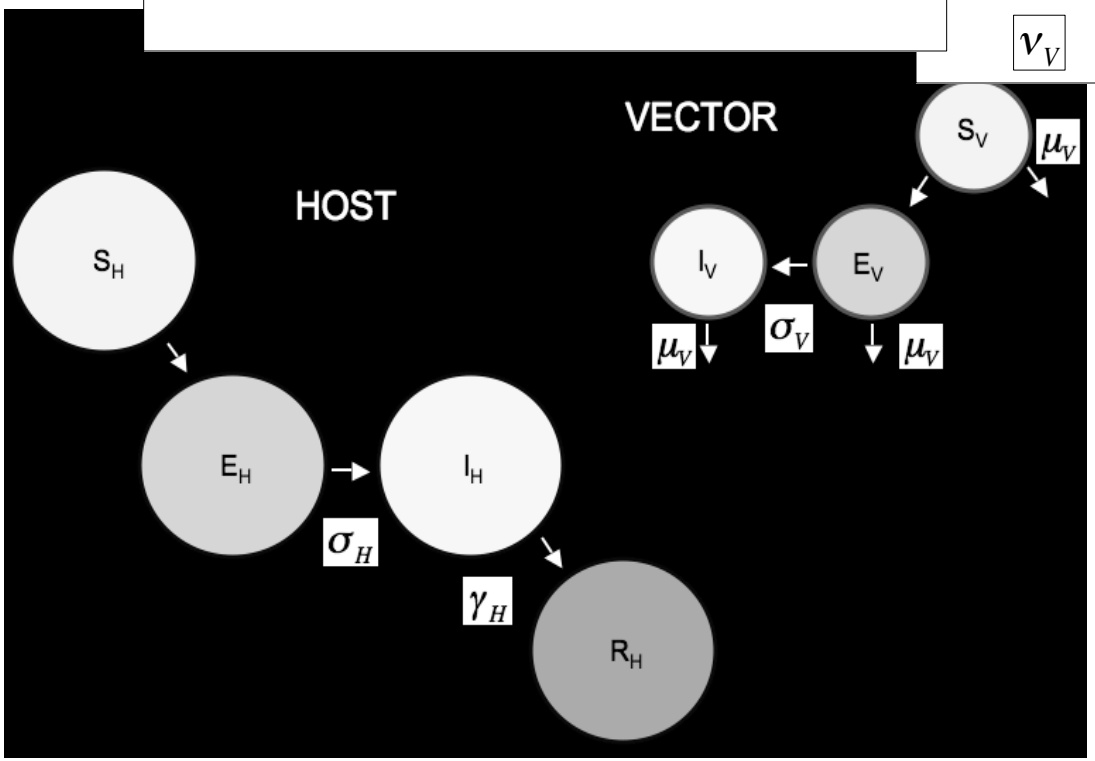


Vector-borne pathogens

not so A simple view of the world

ν_V birth rate

μ_V per capita mortality rate



ν_V per capita birth rate

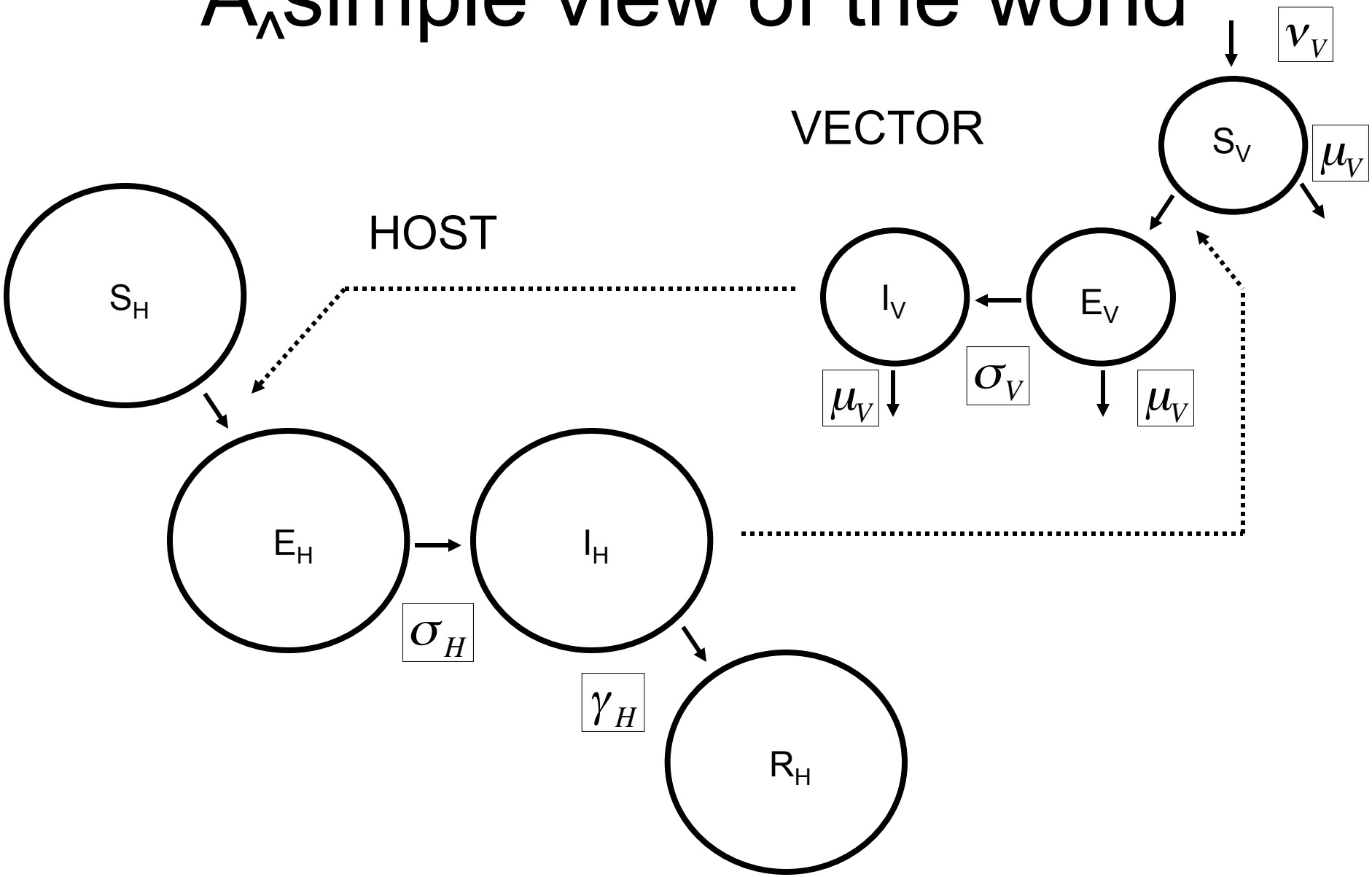
per capita mortality rate

σ_H, σ_V 1/latent period

γ_H 1/infectious period

Vector-borne pathogens

not so A simple view of the world



Vector-borne pathogens

not so A simple view of the world

β = infectivity x per capita contact rate

β = infectivity x per capita (vector) biting rate

infectivity = proportion of susceptible individuals that become infected, given exposure

per capita (vector) biting rate = bites by one individual vector per time unit

HOST

exposure = bite by I_V

VECTOR

exposure = bite on I_H

Vector-borne pathogens

not so A simple view of the world

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infectivity = proportion of susceptible individuals that become infected, given exposure

per capita (vector) biting rate = bites by one individual vector per unit time

HOST

exposure = bite by I_V

infectivity to host = host infections produced per bite by I_V on S_H

β_H = bites (potentially infectious to host) by one individual vector per unit time

$\beta_H I_V$ = bites (potentially infectious to host) per unit time

$\beta_H I_V / N_H$ = bites (potentially infectious to host) per host per unit time

$\beta_H S_H I_V / N_H$ = infectious bites per unit time

Vector-borne pathogens

not so A simple view of the world

β = infectivity x per capita contact rate

β = infectivity x per capita biting rate

infectivity = proportion of susceptible individuals that become infected, given exposure

per capita (vector) biting rate = bites by one individual vector per unit time

VECTOR

exposure = bites on I_H

infectivity to vector = vector infections produced per bite by S_V on I_H

β_V = bites (potentially infectious to vector) by one individual vector per unit time

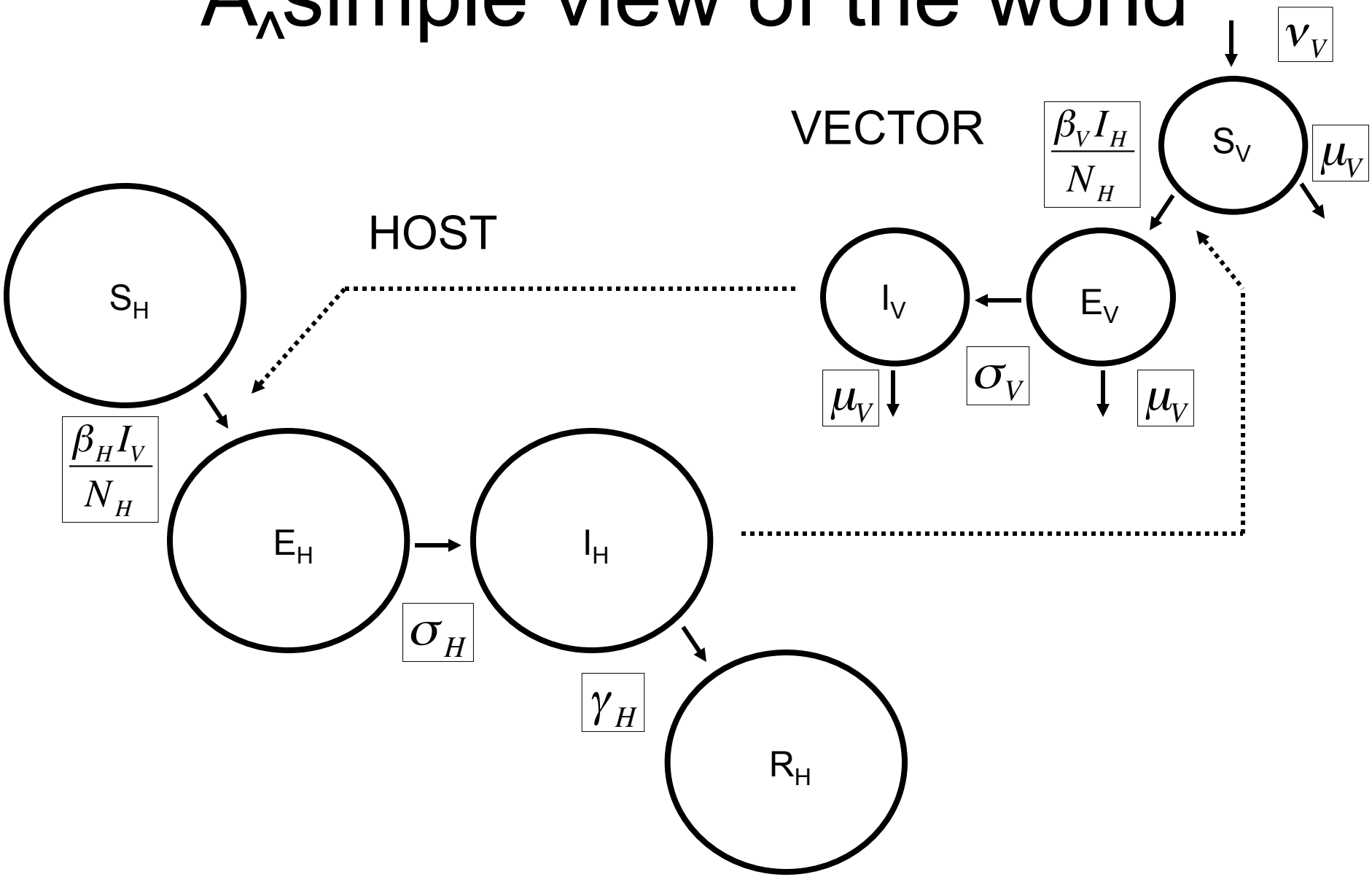
$\beta_V S_V$ = bites (potentially infectious to vector) per unit time

$\beta_V S_V / N_H$ = bites (potentially infectious to vector) per host per unit time

$\beta_V S_V I_H / N_H$ = infectious bites per unit time

Vector-borne pathogens

not so A simple view of the world



Vector-borne pathogens

not so A simple view of the world

HOST

$$\frac{dS_H}{dt} = \frac{-\beta S_H I_V}{N_H}$$

$$\frac{dE_H}{dt} = \frac{\beta S_H I_V}{N_H} - \sigma E_H$$

$$\frac{dI_H}{dt} = \sigma E_H - \gamma I_H$$

$$\frac{dR_H}{dt} = \gamma I_H$$

VECTOR

$$\frac{dS_V}{dt} = \nu_V - \mu_V S_V - \frac{\beta_V S_V I_H}{N_H}$$

$$\frac{dE_V}{dt} = \frac{\beta_V S_V I_H}{N_H} - (\mu_V + \sigma_V) E_V$$

$$\frac{dI_V}{dt} = \sigma_V E_V - (\mu_V + \gamma_V) I_V$$

Vector-borne pathogens

not so

A simple view of the world

$$R_0 = ?$$

HOST

$$\frac{dS_H}{dt} = \frac{-\beta S_H I_V}{N_H}$$

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Vector-borne pathogens

A simple method for complex models

$$R_0 = \rho\left(FV^{-1}\right)$$

FV^{-1} = is the next generation matrix

For all compartments x_i containing infected individuals (ie, E_H, I_H, E_V, I_V), the time derivative can be rewritten as

$$\frac{dx_i}{dt} = f_i(x) = \mathcal{F}_i(x) - \mathcal{V}_i^-(x) + \mathcal{V}_i^+(x)$$

where

$\mathcal{F}_i(x)$ = the rate of appearance of new infections in compartment x_i

$\mathcal{V}_i^-(x)$ = the rate of transfer out of compartment x_i

$\mathcal{V}_i^+(x)$ = the rate of transfer of individuals into compartment x_i, other than new infections

Vector-borne pathogens

A simple method for complex models

$$R_0 = \rho\left(FV^{-1}\right)$$

FV^{-1} = is the next generation matrix

F and V are then the square matrices defined by

$$F = \left[\frac{\partial \mathcal{F}_i(x_0)}{\partial x_j}\right] \quad \text{and} \quad V = \left[\frac{\partial \mathcal{V}_i(x_0)}{\partial x_j}\right]$$

where

$$\mathcal{V}_i = \mathcal{V}_i^- - \mathcal{V}_i^+(x)$$

Vector-borne pathogens

not so A simple view of the world

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For our system, we have

$$x_1 = E_H, x_2 = I_H, x_3 = E_V, x_4 = I_V$$

Vector-borne pathogens

not so A simple view of the world

$$\frac{dS_H}{dt} = \frac{-\beta S_H I_V}{N_H}$$
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$$\frac{dI_V}{dt} = \sigma_V E_V - (\mu_V + \gamma_V) I_V$$

For our system, we have

$$x_1 = E_H, x_2 = I_H, x_3 = E_V, x_4 = I_V$$

and we find

$$F = \begin{bmatrix} 0 & 0 & 0 & \beta_H \\ 0 & 0 & 0 & 0 \\ 0 & \beta_V & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$V = \begin{bmatrix} \sigma_H & 0 & 0 & 0 \\ -\sigma_H & \gamma_H & 0 & 0 \\ 0 & 0 & \mu_V + \sigma_V & 0 \\ 0 & 0 & -\sigma_V & \mu_V + \sigma_V \end{bmatrix}$$

Vector-borne pathogens

not so A simple view of the world

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For our system, we have

$$x_1 = E_H, x_2 = I_H, x_3 = E_V, x_4 = I_V$$

which gives

$$FV^{-1} = \begin{bmatrix} 0 & 0 & \frac{\beta_H \sigma_V}{(\mu_V + \sigma_V)^2} & \frac{\beta_H}{\mu_V + \sigma_V} \\ 0 & 0 & 0 & 0 \\ \frac{\beta_V}{\gamma_H} & \frac{\beta_V}{\gamma_H} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Vector-borne pathogens

not so A simple view of the world

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For our system, we have

$$x_1 = E_H, x_2 = I_H, x_3 = E_V, x_4 = I_V$$

"next generation matrix"

$$FV^{-1} = \begin{bmatrix} 0 & 0 & R_0^{(E_V \to E_H)} & R_0^{(I_V \to E_H)} \\ 0 & 0 & 0 & 0 \\ R_0^{(E_H \to E_V)} & R_0^{(I_H \to E_V)} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Vector-borne pathogens

not so A simple view of the world

$$\frac{dS_H}{dt} = \frac{-\beta S_H I_V}{N_H}$$
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For our system, we have

$$x_1 = E_H, x_2 = I_H, x_3 = E_V, x_4 = I_V$$

and

$$R_0 = \rho\left(FV^{-1}\right) = \sqrt{\frac{\beta_H \beta_V \sigma_H}{(\mu_V + \sigma_V)^2 \gamma_H}}$$

$$R_0^{\ 2} = \frac{\beta_H \beta_V \sigma_H}{(\mu_V + \sigma_V)^2 \gamma_H}$$